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RAND REAC MANUAL

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I. INTRODUCTION

A. Purpose of Manual

The myriad modifications of the RAND REAC by Bill Gunning and associates have necessitated the writing of a manual to replace the obsolete Reeves instructions. Moreover, this manual will permit the recording of RAND computing experiences gained in two years of REAC operation.

The installation of the removable IBM-type plugboard is the major modification made on the REAC and as a consequence will receive the major emphasis.

This manual demands of the reader a knowledge of Kirchhoff's Laws, operational calculus notation, etc. Instructions for the neophyte will appear in the future in a less foreboding form and size than this manual.

B. What is the REAC?

The REAC is an electronic and electromechanical differential analyzer. As the name implies, the machine is primarily intended for the solution of ordinary differential equations, but several other types of equations can be handled equally well.

The differential analyzer is an analogue computer as opposed to a digital computer. An analogue computer plots a continuous curve of a function it is generating, whereas a digital computer, which operates with actual numbers, computes the function only at a finite number of discrete points. In general, a digital machine computes by addition, subtraction, multiplication, and division according to instructions fed into it. An analogue computer depends upon the control of physical quantities, such as electrical voltages or shaft rotations, varied in such a manner as to correspond continually to the variables of the equations or systems being studied.

A major difference between analogue and digital computers lies in their accuracy. In a digital computer the accuracy is determined by the number of significant figures it carries, while the overall accuracy of an analogue device depends upon the accuracy of its individual components and measuring devices. Where more than four significant figures are demanded digital computers must be used, but if two or three significant figures will suffice, analogue computation usually proves to be cheaper and simpler.

There are two types of analogue computers – physical and mathematical. The physical quantities reprsenting the variables in a physical analogue computer are controlled to agree with their original counterparts by designing the components or the computer to operate on the computer variables analogous to the manner in which the physical system operates on its variables. The analogue between dashpots and resistors, springs and condensers, and mass and inductors are familiar to most engineers. While considerable

work has been done on physical analogue models by the communication industry, Dr. McCann of Cal Tech, and others, the major difficulty of such computers lies in synthesizing a proper analogue.

The differential analyzer is a mathematical type of analogue computer. The units of the differential analyzer - the integrator, the adder, the multiplier, etc. - perform the mathematical operations specified by the equations of the system being studied. Hence, instead of striving to find the mechanical or electrical analogue of a system, one need find only the governing equations of a system to prepare it for differential analyzer computation.

Unlike the mathematician, the differential analyzer notices no particular difference between linear and non-linear equations, and wades into implicit equations almost as easily as it does into explicit.

Most differential analyzers may be classified as mechanical or electronic. The mechanical models have been in existence for many years, but their electronic cousins were not developed until World War II. The REAC has approximately ten times the relative inaccuracy of the mechanical models, but is less expensive and bulky, operates about ten times as rapidly, and is far more flexible.

Variables in a mechanical differential analyzer are represented by shaft rotations, while in an electronic model quantities are represented by voltages. The constant of proportionality between the voltage at a point in the circuit and the variable it represents is referred to as the scale factor; it is the number of volts representing one unit of the variable.

REAC problem preparation requires from a few minutes to several weeks depending on the problem complexity, while individual runs require the order of a minute once the plugboard has been wired. However, altering parameters, changing graph paper, recording

results, and other necessary activities limit the output to a maximum of about 20 runs per hour.

Computers operating with a real time scale may be connected to a physical system and act as the analogue of a portion of the system. A computer in such an application is called a simulator.

As a demonstration of the flexibility of the REAC, the following problems are representative of the 70 that have been solved on the RAND REAC.

Examples:

1) Plots of x_1, x_2, x_3, and x_4 versus t were required for the following equations:

$$\dot{x}_1 = (a - bx_1^2 - cx_8^2)F(x_3) - d \sin x_2$$

$$x_1\dot{x}_2 = cF(x_3)x_8x_1 - d \cos x_2$$

$$\dot{x}_3 = x_1 \sin x_2$$

$$\dot{x}_4 = x_1 \cos x_2$$

$$\dot{x}_5 = e(a - bx_1^2 + cx_8^2)F(x_3)x_6$$

$$\dot{x}_6 = x_1x_9 \cos x_2 - x_1x_5 \sin x_2 + 2bF(x_3)x_1^2x_6 - d \frac{x_7}{x_1} \cos x_2$$

$$\dot{x}_7 = x_1x_9 \sin x_2 - x_1x_5 \cos x_2 + dx_6 \cos x_2 - d \frac{x_7}{x_1} \sin x_2$$

$$x_8 = x_7/2x_6$$

$$\dot{x}_9 = 0$$

2) Final values of x_1 through x_6 at t = n and 2n were requested for the following system:

$$\dot{x}_1 = 2x_1(1 - \frac{x_1}{a_1}) - b_1 x_1 \left[1 - \exp(-c_1 x_8)\right]$$

$$\dot{x}_2 = 2x_2(1 - \frac{x_2}{a_2}) - b_2 x_2 \left[1 - \exp(-c_2 x_7)\right]$$

$$\dot{x}_3 = x_9 x_1 - d_3 x_3 \left[1 - \exp(-g_3 \frac{x_5}{x_3})\right] - b_3 x_3 \left[1 - \exp(-c_3 x_8)\right]$$

$$\dot{x}_4 = x_{10} x_2 - d_4 x_4 \left[1 - \exp(-g_4 \frac{x_5}{x_4})\right] - b_4 x_4 \left[1 - \exp(-c_4 x_7)\right]$$

$$\dot{x}_5 = (1 - x_9)x_1 - d_5 x_4 \left[1 - \exp(-g_5 \frac{x_5}{x_4})\right]$$

$$\dot{x}_6 = (1 - x_{10})x_2 - d_6 x_3 \left[1 - \exp(-g_6 \frac{x_6}{x_3})\right]$$

$$x_7 = d_7 \left[a_7 + b_7(c_7 t - e_7)\right] \exp(-g_7 \frac{x_6}{x_3})$$

$$x_8 = d_8 \left[a_8 + b_8(c_8 t - e_8)\right] \exp(-g_8 \frac{x_5}{x_4}$$

$$x_9 = \frac{a_9 x_5 x_6}{a_9 x_5 x_6 + x_3 x_4}$$

$$x_{10} = \frac{a_{10} x_5 x_6}{a_{10} x_5 x_6 + x_3 x_4}$$

3) The following problem is typical of a class in which no differential equations are involved and the REAC becomes merely an equation solver. Plots were required of x_1 and x_2 versus x_3, where

$$x_1 = \text{arc sin} (a \frac{x_5}{x_6} \sin b) - c$$

$$x_2 = \frac{m x_6 \cos (x_1 + x_4)}{n(a \sin b - \bar{n} \sin (x_1 + \gamma)}$$

$$x_3 = m x_6 \cos (x_1 + x_4) - a x_5 \cos b$$

$$\tan x_4 = \frac{a \sin \gamma}{p + \bar{n} \cos \gamma}$$

$$x_5 = F_1(x_7) - F_1(x_0)$$

$$x_6 = F_2(x_7) - F_2(x_0)$$

4) The following problem is a special case of solving for the roots of an implicit function $F(x) = 0$. Find the real roots of the quartic:

$$ax^4 + bx^3 + cx^2 + dx + e = 0.$$

5) The following problem required modification before it could be solved on the REAC. The equation

$$x = \int_0^v \left[a_1 + b_1(1 - e^{-\beta_1 t}) \right] e^{-rt} dt + c(1 - c_1 e^{-\lambda_1 v}) e^{-rv}$$

$$+ c(1 - K_2 e^{-\lambda n}) \sum_{j=1}^{\infty} e^{-r(v+jn)} + \sum_{j=0}^{\infty} \int_0^n \left[ae^{-\alpha(v+jn)} \right.$$

$$+ b(1 - e^{-\beta \tau}) \gamma^{nj} \left] e^{-r(v+jn+\tau)} d\tau - \int_0^v (p_1 e^{-\phi_1 t}) e^{-rt} dt \right.$$

$$- \sum_{j=0}^{\infty} \int_0^n \left\{ p + q\left[1 - e^{-\theta(v+jn)} \right] \right\} e^{-\phi \tau - r(v+jn+\tau)} dt$$

became more suitable for REAC computation by eliminating the summation signs to yield

$$x = \int_0^v \left[a_1 + b_1(1 - e^{-\beta_1 t}) \right] e^{-rt} dt + c(1 - c_1 e^{-\lambda_1 v}) e^{-rv}$$

$$+ \frac{c(1 - K_2 e^{-\lambda n} - e^{-r(v+n)})}{1 - e^{-rn}} + \frac{ae^{-v(r+\alpha)}(1 - e^{-rn})}{r(1 - e^{-(r+\alpha)n})}$$

$$+ be^{-rv}\left[\frac{(1 - e^{-rn})}{r(1 - \gamma^n e^{-rn})} - \frac{(1 - e^{-(\beta+r)n})}{(\beta + r)(1 - \gamma^n e^{-rn})} \right] - p_1 \left\{ \frac{1 - e^{-(\phi_1+r)v}}{\phi_1 + r} \right\}$$

$$- \left[\frac{p+q}{1-e^{-rn}} - \frac{qe^{-\theta v}}{1-e^{-n(\theta+r)}} \right] \left[\frac{1-e^{-(\phi+r)n}}{\phi+r} \right] e^{-rv}$$

Other examples will appear throughout the manual in demonstration of various techniques.

II. MAJOR COMPONENTS

A. D. C. Amplifiers

The most important component of the REAC is the d.c. amplifier, which is the basis of adders, integrators, etc. The input and feedback impedances of the amplifier determine the mathematical operation it performs on the input signal.

1. Summing Amplifier

If the input and feedback elements of an amplifier are resistors, the output will be the sum of the inputs with an inversion in sign and possible multiplication factors as determined by the ratio of the feedback resistor to the input resistors.

The following equations, using the notation of Figure 1, illustrate the influence of the input and feedback resistors:

$$i_1 = \frac{e_i - e_{in}}{R_1}$$

$$i_f = \frac{e_{in} - e_o}{R_f}$$

$i_1 = i_f$, assuming no grid current.

Since $\mu = -\dfrac{e_o}{e_{in}}$ by definition,

$$\frac{e_i + \dfrac{e_o}{\mu}}{R_1} = -\frac{\dfrac{e_o}{\mu} + e_o}{R_f} = -\frac{e_o}{R_f}\left(1 + \frac{1}{\mu}\right)$$

$$\text{or } e_o = -e_i \frac{\dfrac{R_f}{R_i}}{1 + \dfrac{1}{\mu}\left(1 + \dfrac{R_f}{R_1}\right)} \doteq -e_i \frac{R_f}{R_1} \text{ for large } \mu.$$
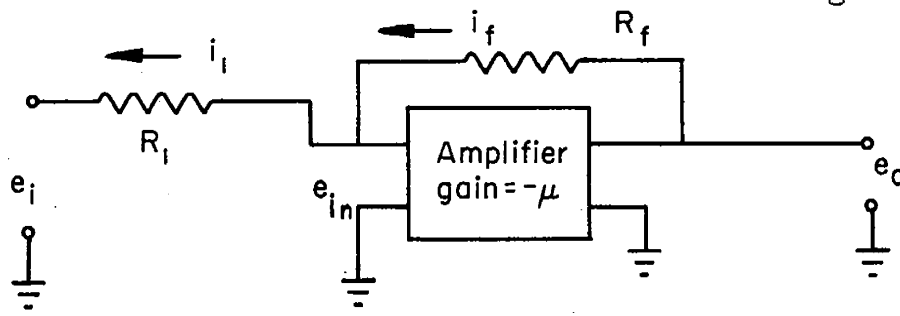
Fig. 1 — Basic amplifier circuit
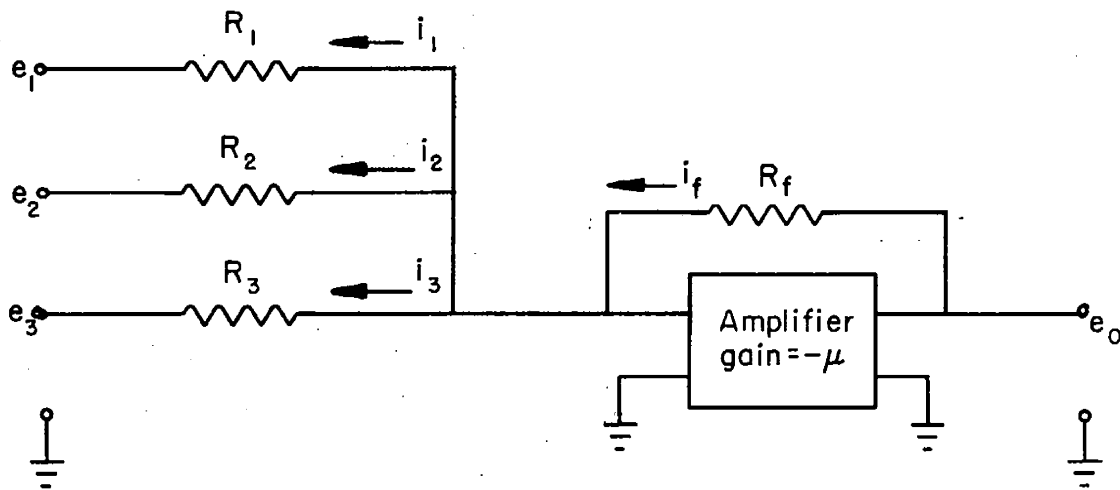
Fig. 2 — Summing amplifier
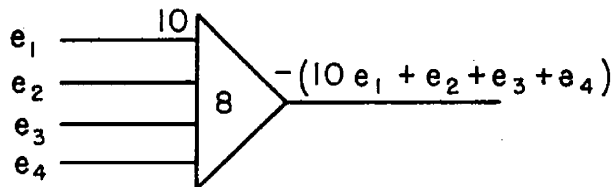
Fig. 3 — Summing amplifier schematic diagram

Similarly,

$$i_1 = \frac{e_i}{R_1}\left[1 - \frac{1}{\mu}\frac{R_f}{R_1}\right] \doteq \frac{e_i}{R_1}$$

$$i_f = \frac{e_o}{R_f}\left(1 + \frac{1}{\mu}\right) \doteq -\frac{e_o}{R_f}$$

From this analysis, it follows that if there are several inputs, as shown in Figure 2, the relationships are:

$$i_1 + i_2 + i_3 = i_f$$

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} \doteq -\frac{e_o}{R_f}$$

$$e_o \doteq -\left(\frac{R_f}{R_1}e_1 + \frac{R_f}{R_2}e_2 + \frac{R_f}{R_3}e_3\right)$$

The resistance ratio, $\frac{R_f}{R_1}$ for example, is called the gain or scale factor of that particular input. Usually the input resistance equals the feedback resistance and the corresponding gain is unity. The basic resistors in the REAC have one megohm resistance.

The schematic diagram for a summing amplifier is illustrated in Figure 3. The ground terminals are omitted for simplification, and the number of the amplifier to be used is placed within the triangle. Unless otherwise noted, input gains are unity.

2. Integrating Amplifier

If the feedback resistor is replaced by a condenser the amplifier becomes an integrator. In this case, referring to Figure 4,
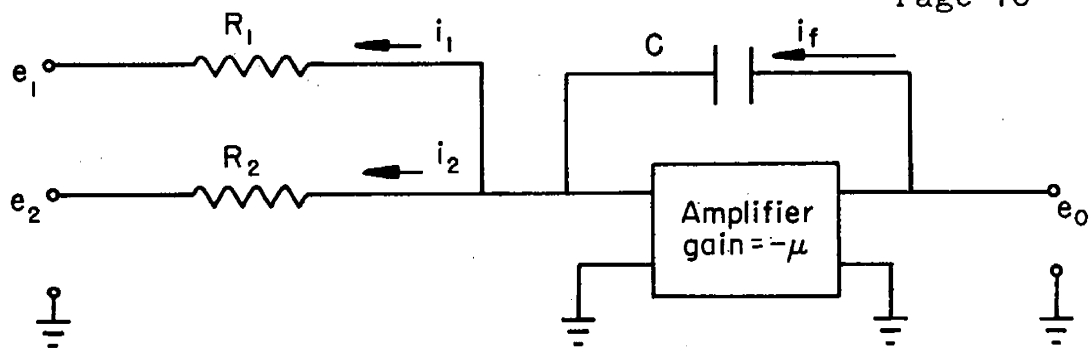
Fig. 4 — Integrating amplifier
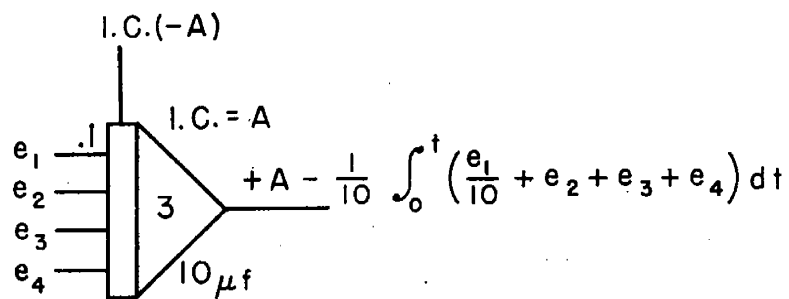
Fig. 5 — Integrating amplifier schematic

$$+A - \frac{1}{10} \int_0^t \left(\frac{e_1}{10} + e_2 + e_3 + e_4 \right) dt$$

$$e_0 \doteq - \frac{Z_f(p)}{Z_i(p)} e_i$$
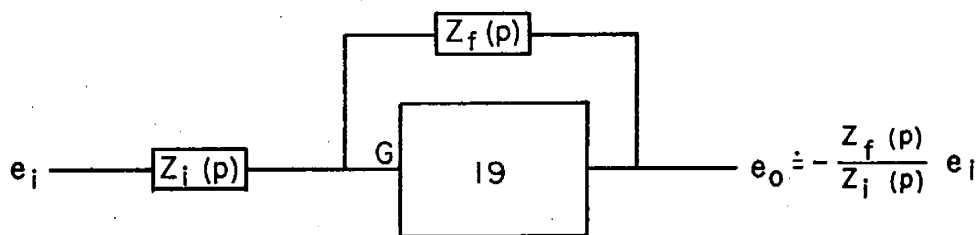
Fig. 6 — Operational amplifier

$$i_1 + i_2 = i_f$$

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} \doteq - \frac{C de_0}{dt}$$

$$e_0 \doteq - \left(\frac{1}{R_1 C} \int_0^t e_1 dt + \frac{1}{R_2 C} \int_0^t e_2 dt \right)$$

The gain of an integrator is $1/RC$. It is preferable to reduce the gain by increasing the size of C rather than R to reduce drift effects (the improved drift stability of the new chopper-type amplifiers reduces the importance of this technique). The standard integrator has a one microfarad feedback condenser, but provision has been made in the RAND REAC for adding condensers in groups of 1, 9, 10, and 20 microfarads to the basic condenser. Condensers are added by connecting them in parallel.

The schematic diagram for an integrator is given in Figure 5. The gain of the input is computed on the basis of a one microfarad feedback condenser (gain $= \frac{1}{R}$, with R in megohms) and is unity (R = one megohm) unless otherwise indicated. Under the integrator is noted the total number of microfarads in the feedback if different than unity. The overall gain is the input gain divided by the number of microfarads in the feedback element. The initial condition voltage (I.C.) is shown entering at the top of the integrator and if the I.C. is constant throughout the problem its output value is noted above the integrator.

3. Operational Amplifier

Input and feedback impedances other than those mentioned above may be employed, and an amplifier so connected will be called an operational amplifier. Using operational notation and the symbols of Figure 6 it can be shown that

$$e_0 = - \frac{Z_f(p)}{Z_i(p)} e_i \left(\frac{1}{1 + \frac{1}{\mu}\left(1 + \frac{Z_f(p)}{Z_i(p)}\right)} \right) \doteq - \frac{Z_f(p)}{Z_i(p)} e_i$$

For example, if $Z_f(p) = R$ and $Z_i(p) = 1/Cp$

$$e_0 \doteq - RCpe_i = - RC \frac{de_i}{dt} ,$$

and we have a differentiator. Integrating and summing amplifiers are special cases of operational amplifiers. Table I lists several operational amplifiers.

Operational amplifiers are not as useful as they may appear at first sight, primarily because of the difficulty of adjusting the impedances to satisfy the parameters of the problem. For example, assume it is required to form

$$e_0 = - \frac{(1 + ap)e_i}{b(1 + cp)p}$$

and operational amplifier 18 is selected. The four impedances must satisfy the equations
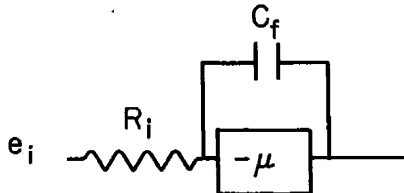
$$(R_i + \overline{R}_i)C_i = a$$
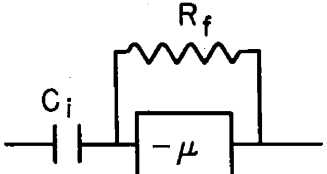
$$\overline{R}_i C_f = b$$
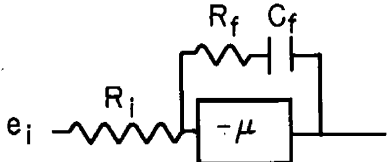
$$R_i C_i = c.$$
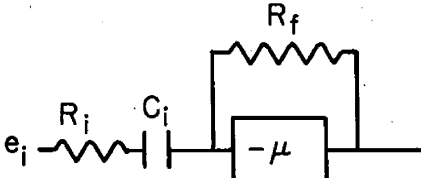
If a, b, or c should vary from run to run it probably would be far simpler to reduce the transform to its differential equation equivalent and use standard differential analyzer techniques for the solution. Moreover, trouble with noise and stability is apt to arise from the use of amplifiers having a higher order of p in the numerator than the denominator of the transform function.
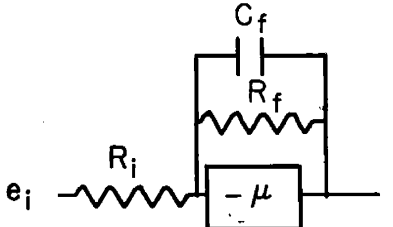
Table I Several operational amplifiers
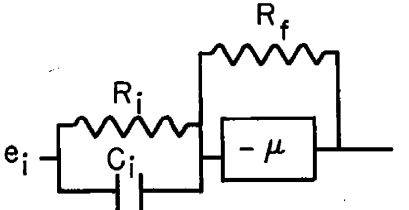
1. e_i ——R_i——[$-\mu$] with C_f in feedback—— $e_o = -\dfrac{1}{R_i C_f p} e_i$

2. e_i ——C_i——[$-\mu$] with R_f in feedback—— $e_o = -R_f C_i p\, e_i$

3. e_i ——R_i——[$-\mu$] with $R_f\ C_f$ in feedback—— $e_o = -\dfrac{\left(1 + R_f C_f p\right)}{R_i C_f p} e_i$

4. e_i ——R_i——C_i——[$-\mu$] with R_f in feedback—— $e_o = -\dfrac{R_f C_i p}{1 + R_i C_i p} e_i$

5. e_i ——R_i——[$-\mu$] with C_f over R_f in feedback—— $e_o = -\dfrac{R_f}{R_i\left(1 + R_f C_f p\right)} e_i$
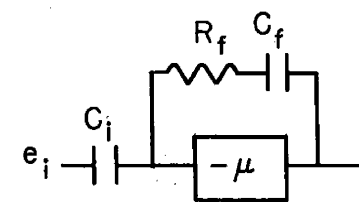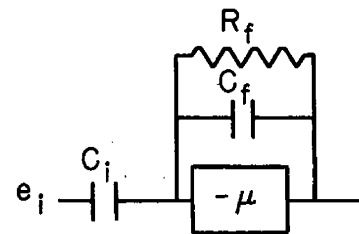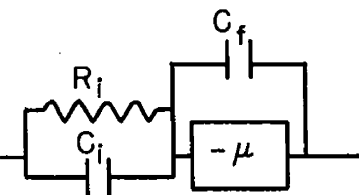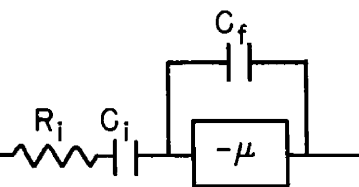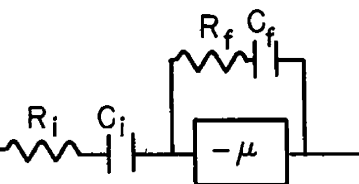
6. e_i ——R_i, C_i——[$-\mu$] with R_f in feedback—— $e_o = -\dfrac{R_f}{R_i}\left(1 + R_i C_i p\right) e_i$
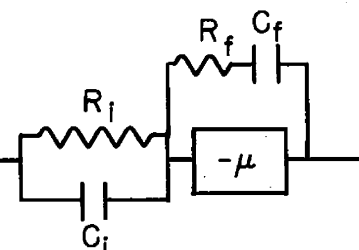
7. $\quad e_o = -\dfrac{C_i}{C_f}\left(1 + R_f C_f p\right) e_i$
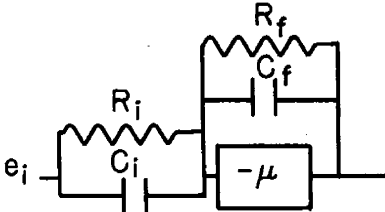
8. $\quad e_o = -\dfrac{R_f C_i p}{1 + R_f C_f p}\, e_i$
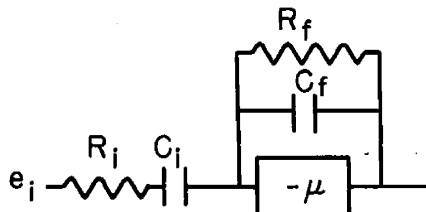
9. $\quad e_o = -\dfrac{1 + R_i C_i p}{R_i C_f p}\, e_i$
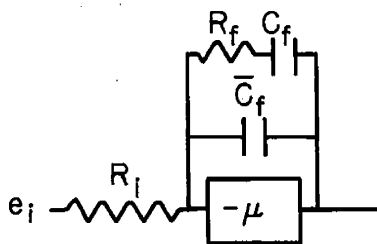
10. $\quad e_o = -\dfrac{C_i}{C_f\left(1 + R_i C_i p\right)}\, e_i$
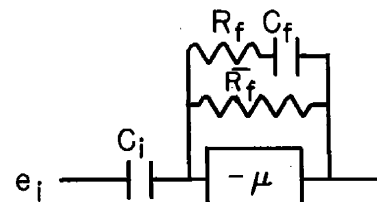
11. $\quad e_o = -\dfrac{C_i}{C_f}\dfrac{\left(1 + R_f C_f p\right)}{\left(1 + R_i C_i p\right)}\, e_i$
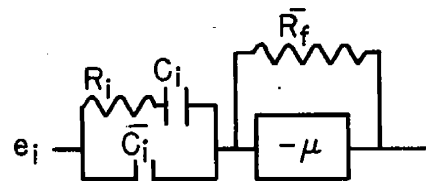
12. $\quad e_o = -\dfrac{\left(1 + R_i C_i p\right)\left(1 + R_f C_f p\right)}{R_i C_f p}\, e_i$
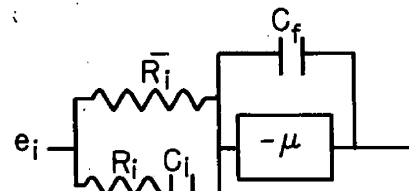
13. $\quad e_0 = -\dfrac{R_f}{R_i}\dfrac{(1+R_iC_ip)}{(1+R_fC_fp)}\,e_i$

14. $\quad e_0 = -\dfrac{R_fC_ip}{(1+R_fC_fp)(1+R_iC_ip)}\,e_i$

15. $\quad e_i = -\dfrac{1+R_fC_fp}{R_i(C_f+\bar{C}_f)\,p\left(1+\dfrac{R_f\,\bar{C}_f\,C_f}{C_f\,\bar{C}_f}\,p\right)}\,e_i$

16. $\quad e_0 = -\dfrac{\bar{R}_f C_i p\,(1+R_fC_fp)}{1+(R_f+\bar{R}_f)\,C_fp}\,e_i$

17. $\quad e_0 = -\dfrac{R_f(C_i+\bar{C}_i)\,p\left(1+\dfrac{R_iC_i\bar{C}_ip}{C_i+\bar{C}_i}\right)}{1+R_iC_ip}\,e_i$

18. $\quad e_0 = -\dfrac{\left[1+(R_i+\bar{R}_i)\,C_ip\right]}{\bar{R}_i\,C_fp\,(1+R_iC_ip)}\,e_i$

A modified operational amplifier was required when it was desired to use the REAC as a frequency spectrum analyzer by feeding a signal into a simulated RLC band-pass filter having a variable frequency f_0 and a band-width Δf. The operational equation for this application is

$$e_0 = \frac{-pe_i}{\frac{1}{C} + Rp + Lp^2}$$

with

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Delta f = f_0/Q$$

$$Q = \frac{2\pi f L}{R}$$

An analysis of operational amplifier 14 indicated that it could yield such an operation, but could not have a value of Q greater than that giving critical damping ($Q = \frac{1}{2}$). The modified operational amplifier of Figure 7 will produce a Q of any value. Assuming no grid current and infinite gain for amplifier No. 1, it can be seen that the system is solving the implicit function

$$\left(\frac{1}{Cp} + R + Lp\right)e_0 + e_i = 0.$$

4. High Gain Amplifier

A summing or integrating amplifier with its feedback loop opened is called a high gain amplifier. The schematic diagram of such an amplifier is illustrated in Figure 8; the input resistors are all one megohm unless otherwise noted. The output of a high gain amplifier is minus μ (the amplifier gain) times the sum of the inputs.

Limiter

.25

20μ

Ave. Amplitude Meter

L

R⁻¹

G

e_i

I

$e_o = - \dfrac{pe_i}{\frac{1}{C} + Rp + Lp^2}$

⅟c

2

$\dfrac{1}{Cp} e_o$

9

(a) Reac circuit

A.C. Ammeter

L

e

i

C

R

$i = \dfrac{pe}{\frac{1}{C} + Rp + Lp^2}$
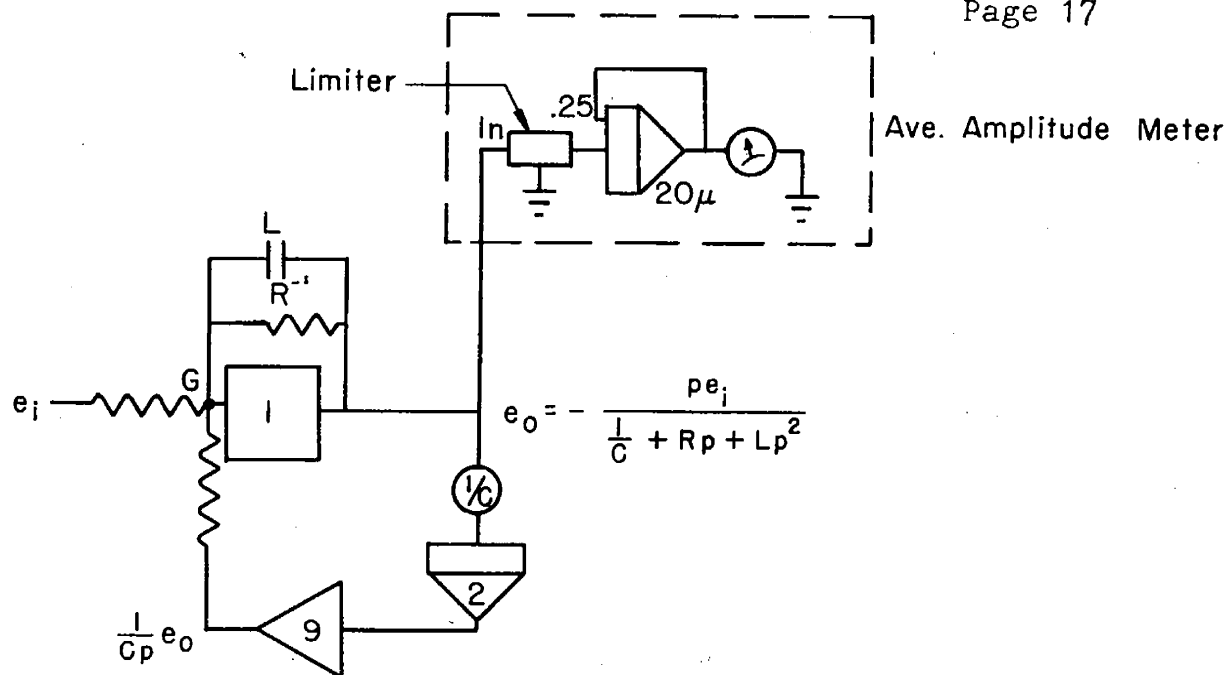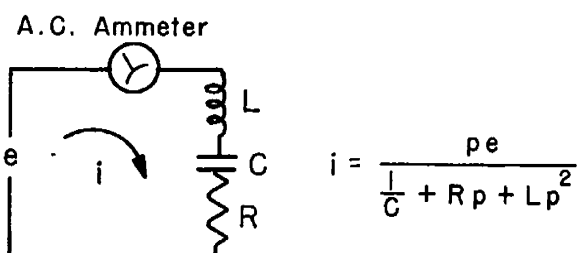
(b) Analogous RLC network

Fig. 7 — Modified operational amplifier used
as a unit of a frequency spectrum analyzer
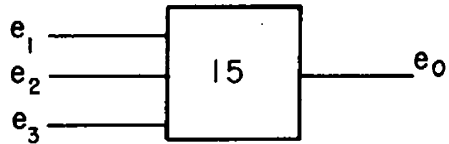
Fig. 8—High gain amplifier schematic
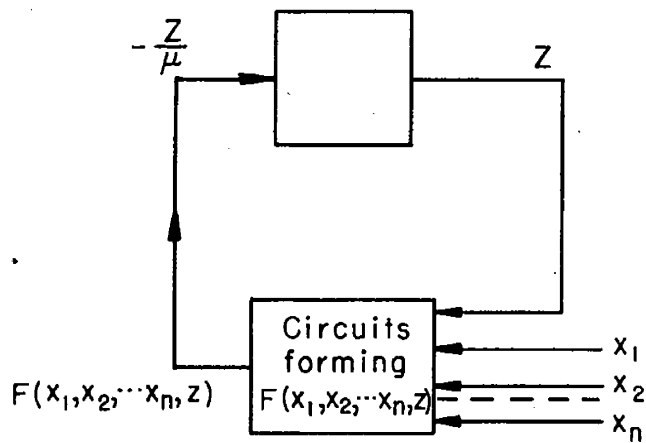
Fig. 9—Use of a high gain amplifier
to solve an implicit equation

$$e_0 = -\frac{1}{k}(e_1 + e_2)$$
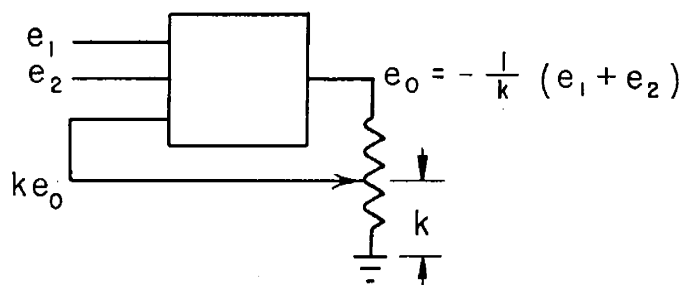
Fig. 10—Adjustable gain amplifier

High gain amplifiers are employed most frequently in implicit function techniques. For example, assume we have a function $z = Z(x_1, x_2, \ldots, x_n)$ in which Z is either difficult or impossible to compute, while an equivalent implicit function $F(x_1, x_2, \ldots, x_n, z) = 0$ offers no such complication. Examples of such functions are $z = \frac{x_1}{x_2}$ and $z \cdot x_2 - x_1 = 0$; $z = \sqrt{x_1^2 + x_2^2}$ and $z^2 - x_1^2 - x_2^2 = 0$; $z = \tan^{-1} \frac{x_1}{x_2}$ and $x_2 \sin z - x_1 \cos z = 0$; and $z = \frac{dx_1}{dt}$ and $x_1 - \int z dt = 0$. If F is fed into a high gain amplifier whose output is used as z in the computation of F, it can be seen from Figure 9 that the feedback amplifier now has a feedback loop closed through the circuits generating F. It is also apparent that the system is actually setting F equal to minus $\frac{z}{\mu}$ instead of zero, but since the magnitude of μ at low frequencies is greater than 10^7, the resulting error is completely negligible.

If a fraction k of the output of a high gain amplifier is fed back into the input at unity gain, the output will be 1/k times the sum of the input signals (see Figure 10). Notice that the implicit equation being solved is

$$\sum e_i + k e_0 = 0$$

When k = 1 the usual summing amplifier results.

5. Relays and Initial Condition Settings

When a d.c. amplifier is used as an integrator, relay circuits must be employed to prevent integration of the input signals until the operate button is pushed, to permit the intro-

duction of initial condition voltages across the feedback condensers, and to permit the operator to stop (or hold) integration once it has been started. Figure 11 is a block diagram of the amplifier circuits and illustrates the relay circuits and other junctions available from the plugboard.

It is sometimes necessary to operate amplifiers without the chopper drift-stabilizing circuits and switches opening the chopper loop are on each amplifier. Without the chopper circuit it is necessary occasionally to hand balance the amplifier. The reset relay is energized and any I.C. voltage is removed for this operation.

6. Boost

When the output tube of an amplifier is cut-off, the voltage reaches its maximum value and is equivalent to a +300 volt power source with an internal resistance of 18K (the plate load resistor of the output tube). When the 20K resistance of a potentiometer is placed across the output, the resultant voltage divider action limits the magnitude of the output voltage. Boost circuits are provided to place 18K in parallel with the source impedance. Table II lists the number of boosts required to obtain various output voltages with different potentiometer loadings. If plus and minus voltages are connected across a servo multiplying potentiometer each amplifier is loaded by the equivalent of two potentiometers.

When a tube is driven to its maximum negative value it appears to the output as a -190 volt source with about a 1K internal resistance. Hence, negative output voltages create no loading problem.

Overload lights are connected to each amplifier to indicate loading or other errors causing the grid voltage to be greater than a millivolt.

(a) Basic amplifier

(b) Normal integrator connections
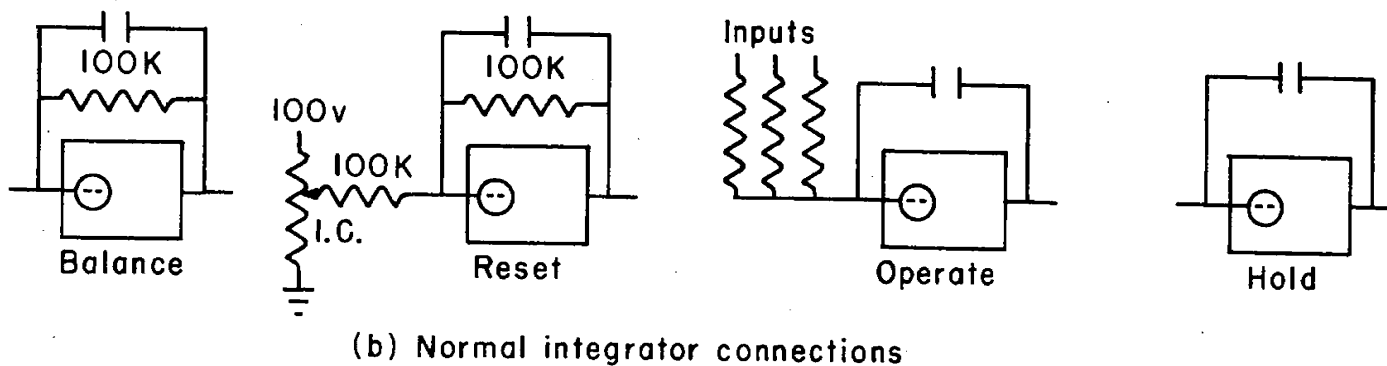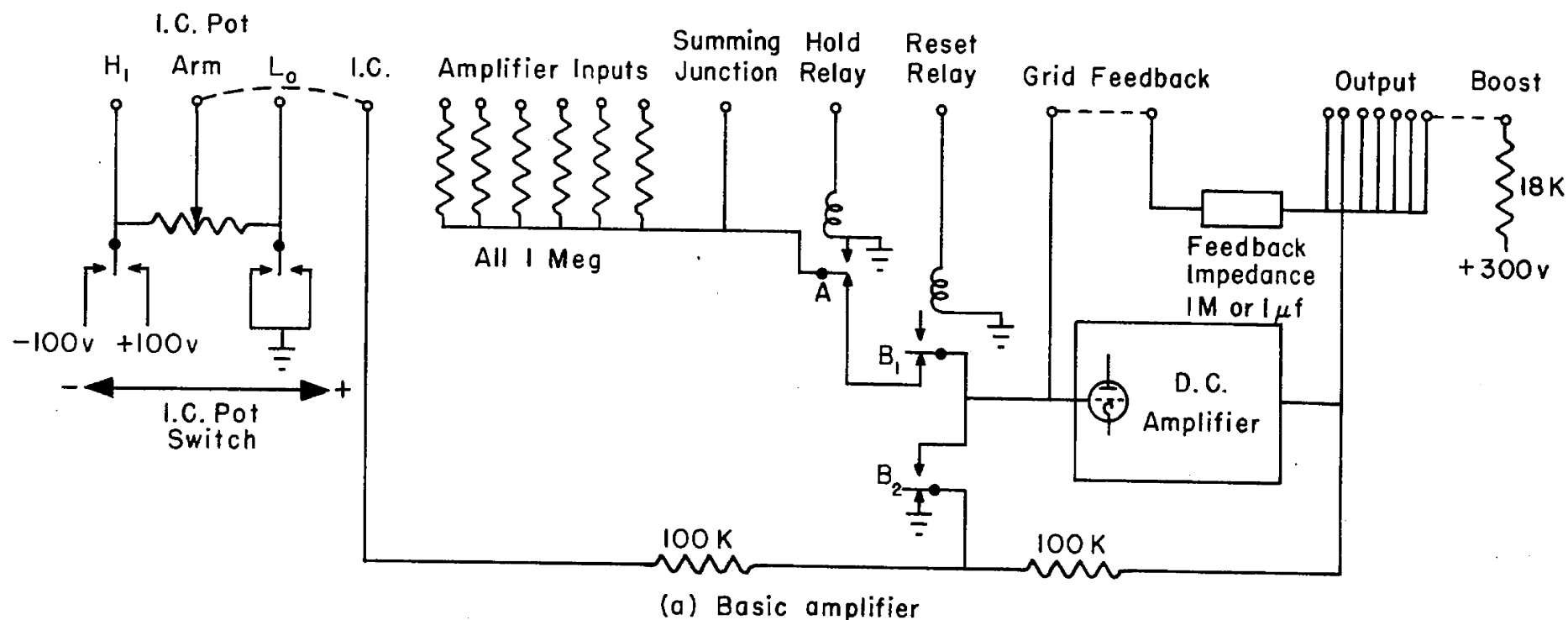
Fig. 11 — Amplifier block diagram

TABLE II. BOOST CONDITIONS

Number of 20K Potentiometers*	Number of Boosts	Approximate Maximum Positive Output Voltage
0	0	300
1	0	158
2	0	107
2	1	158
3	0	82
3	1	128
4	0	65
4	1	107
4	2	136
5	0	54
5	1	92
5	2	120
6	0	48
6	1	81
6	2	108
6	3	128
10	0	30
10	1	55
10	2	75
10	3	92
10	4	107

*When a servo multiplying potentiometer is used
with equal and opposite voltages on its ends,
the loading is equivalent to two 20K potentiometers.

B.   R-C Elements

Extra banks of condensers and resistors are available primarily for use as additional input or feedback elements. Also, they may be used to construct passive networks to operate on signals when there is a shortage of amplifiers. Table III lists several networks compiled by the Reeves Computing Staff. The passive network should feed into an impedance much larger than the network impedances. Since this would ordinarily require abnormally high values of capacitance, it is usually necessary to have the passive network feed into a voltmeter servo or the unloading circuit below which has an equivalent input impedance of over 100 megohms.
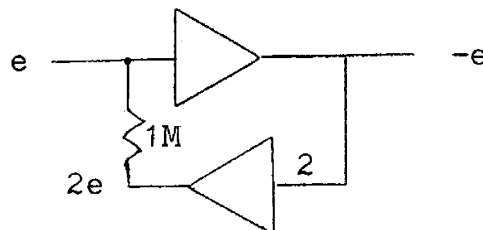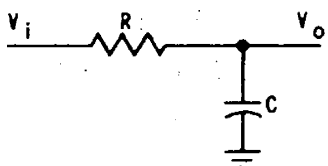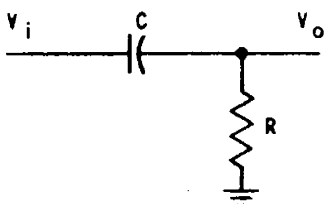
Table III
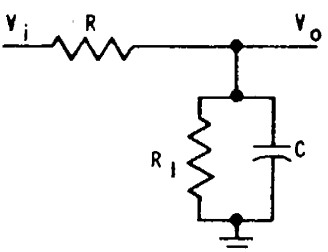
REAC COMPUTER
SUGGESTED PASSIVE NETWORKS

1.

$$\frac{V_o}{V_i} = \frac{1}{1 + RCp} = \frac{1}{1 + ap}$$

2.

$$\frac{V_o}{V_i} = \frac{RCp}{1 + RCp} = \frac{ap}{1 + ap}$$

3.

$$\frac{V_o}{V_i} = \frac{1 + R_1Cp}{1 + (R + R_1)Cp} = \frac{1 + ap}{1 + bp}$$

4.

$$\frac{V_o}{V_i} = \frac{R_1Cp}{1 + (R + R_1)Cp} = \frac{ap}{1 + bp}$$

5.

$$\frac{V_o}{V_i} = \frac{R_1}{(R + R_1) + RR_1Cp} = \frac{a}{b + cp}$$

6.
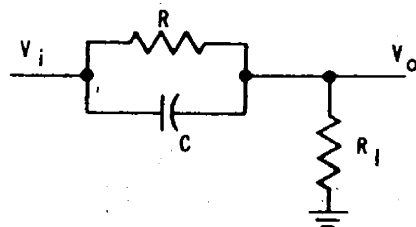
$$\frac{V_o}{V_i} = \frac{R_1 + RR_1Cp}{(R + R_1) + RR_1Cp} = \frac{a + bp}{c + bp}$$

7.
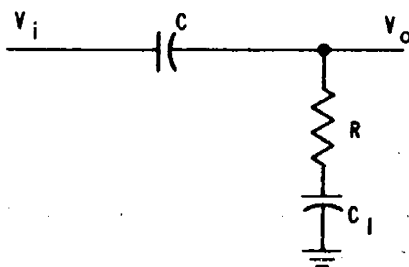
$$\frac{V_o}{V_i} = \frac{C + RCC_1p}{(C + C_1) + RCC_1p} = \frac{a + bp}{c + bp}$$

8.

$$\frac{V_o}{V_i} = \frac{RCp}{1 + R(C + C_1)p} = \frac{ap}{1 + bp}$$

9.

$$\frac{V_o}{V_i} = \frac{1 + RCp}{1 + R(C + C_1)p} = \frac{1 + ap}{1 + bp}$$

10

$$\frac{V_o}{V_i} = \frac{C}{(C + C_1) + RCC_1p} = \frac{a}{b + cp}$$
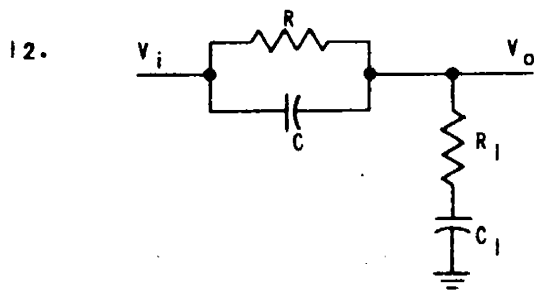
11.

$$\frac{V_o}{V_i} = \frac{C + R_1CC_1p}{(C + C_1) + (R + R_1)CC_1p} = \frac{a + bp}{c + dp}$$

12.

$$\frac{V_o}{V_i} = \frac{1 + (CR + C_1R_1)p + RR_1CC_1p^2}{1 + (CR + C_1R_1 + C_1R)p + RR_1CC_1p^2}$$

$$= \frac{1 + ap + bp^2}{1 + cp + bp^2}$$

13.

$$\frac{V_o}{V_i} = \frac{R_1 + RR_1Cp}{(R + R_1) + RR_1(C + C_1)p} = \frac{a + bp}{c + dp}$$

14.

$$\frac{V_o}{V_i} = \frac{R_1Cp}{1 + (RC + R_1C_1 + R_1C)p + RR_1CC_1p^2}$$

$$= \frac{ap}{1 + bp + cp^2}$$

15.

$$\frac{V_o}{V_i} = \frac{1 + R_1Cp}{1 + (RC + R_1C + RC_1)p + RR_1CC_1p^2}$$

$$= \frac{1 + ap}{1 + bp + cp^2}$$

16.
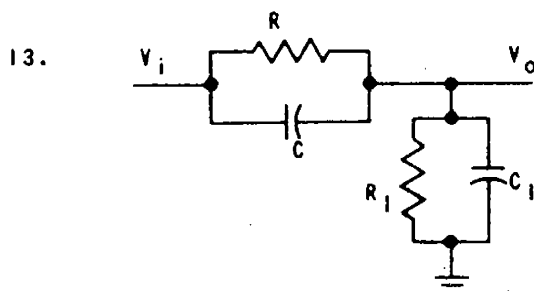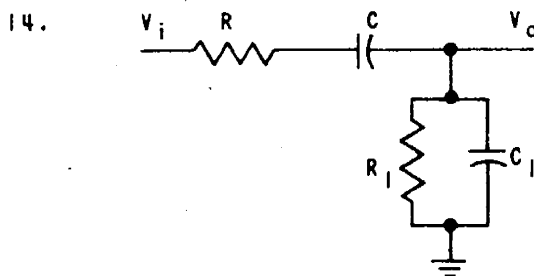
$$\frac{V_o}{V_i} = \frac{RCp + RR_1CC_1p^2}{1 + (RC + R_1C_1 + RC_1)p + RR_1CC_1p^2}$$

$$= \frac{ap + bp^2}{1 + cp + bp^2}$$

17.

$$\frac{V_o}{V_i} = \frac{R_1(C + C_1)p + RR_1CC_1p^2}{1 + (RC + R_1C_1 + R_1C)p + RR_1CC_1p^2}$$

$$= \frac{ap + bp^2}{1 + cp + bp^2}$$

18.
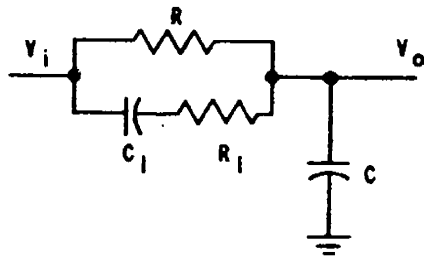
$$\frac{V_o}{V_i} = \frac{1 + (R + R_1)C_1p}{1 + (RC + R_1C_1 + RC_1)p + RR_1CC_1p^2}$$

$$= \frac{1 + ap}{1 + bp + cp^2}$$

C. Multipliers

Since only a limited number of gains are possible by varying input or feedback impedances, other means of modifying scale factors are necessary.

1. Constant Factor Potentiometers

Multiplication by a constant is accomplished by hand-positioned constant factor potentiometers, diagrammed schematically in Figure 12. It is obvious that these potentiometers can multiply only by a fraction. The component number of the potentiometer is placed within the circle and the setting is placed next to the circle.

2. Multiplying Servos

Potentiometers, similar to the scale factor potentiometers, are used for multiplication by a variable. However, instead of being positioned by hand these potentiometers are positioned continuously by a variable or the sum of variables. Each servo motor is mechanically coupled to three multiplying potentiometers and one "follow-up" potentiometer. Figure 13 is the schematic notation for such a system. The servo type ($\sum$ for summing and V for voltmeter) is indicated within the circle.

In a summing servo, the sum of the inputs, $\sum e_i$, plus the voltage of the mechanically-coupled follow-up potentiometer arm, e_f, is fed to an amplifier that drives the servo motor until $\sum e_i + e_f = 0$. Since $e_f = 100k$ volts, the follow-up potentiometer has a setting

$$k = -\frac{\sum e_i}{100}$$

as do the multiplying potentiometers, which are also mechanically coupled to the motor. Hence the products result as indicated in the diagram.
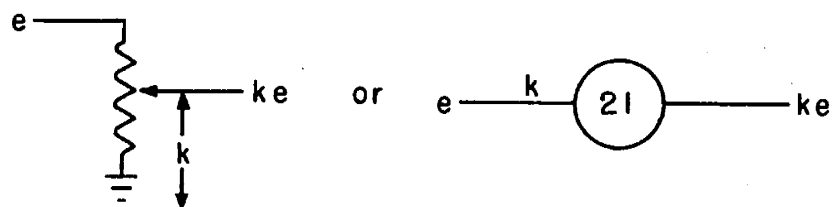
Fig. 12—Scale factor potentiometer schematic

$$-\frac{(e_1+e_2+e_3)}{100}r \qquad +\frac{(e_1+e_2+e_3)}{100}s \qquad +\frac{\left[\frac{(e_1+e_2+e_3)}{100}+1\right]t}{2}$$
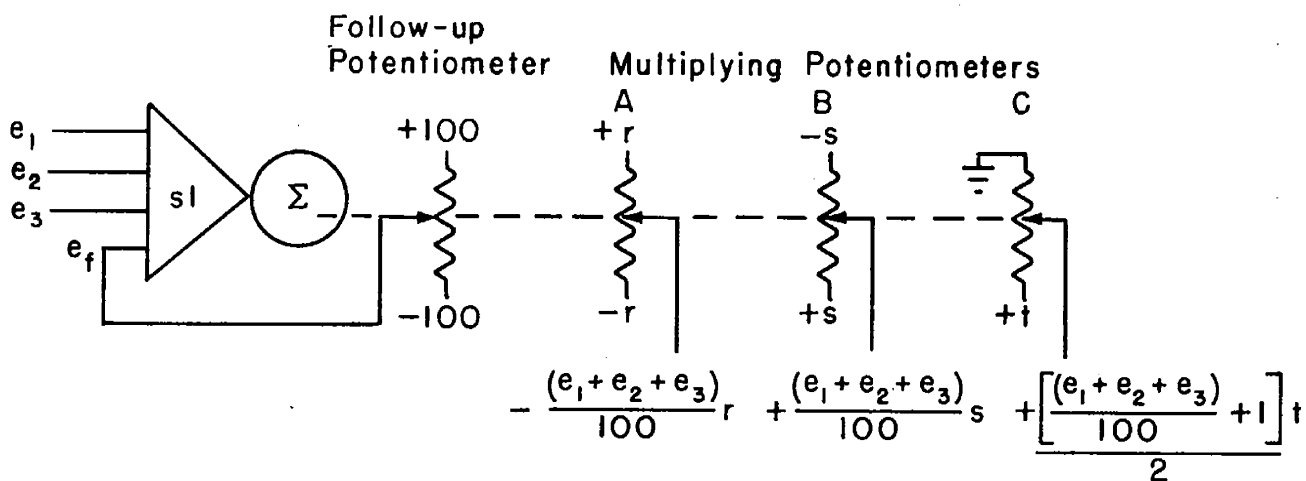
Fig. 13—Multiplying servo schematic

The indication on the servo dial is the negative of the sum of the input signals except for the effect of the load of a one megohm feedback resistor on the follow-up potentiometer. In order to produce a voltage at the follow-up potentiometer arm equal in magnitude but opposite in sign to the sum of the input signals, the servo motor must turn beyond the actual value of 100k percent. Hence, the dial will indicate a value slightly larger in magnitude than the correct sum. The correction is less than 0.13 percent when a center-tapped follow-up potentiometer is used. This mode of operation will produce a correct product if the multiplying potentiometers also are loaded with the customary one megohm. As a consequence, the arm of every potentiometer must go to a d.c. or servo amplifier and never to another potentiometer.

Plugboard switching (see Figure 14) allows the circuit to be restored to a positive gain servo. In this "voltmeter" mode of operation the equivalent loading impedance seen by the source driving the servo at the secondary input is very high. Similarly, the follow-up potentiometer is effectively unloaded. Consequently, the dial indication is as accurate as the follow-up potentiometer, and is of the same sign as the input voltage.

The off-balance input impedance of the servo is of the order of one megohm in either voltmeter or summing position. This allows the servo to be switched from point to point during a solution without appreciably affecting the signals to be observed.

As can be seen from an inspection of Figure 14, the voltages across the follow-up potentiometer may be altered or the follow-arm feedback loop opened in which case the servo acts in the same manner as a high gain amplifier.

Fig. 14—Servo block diagram

Secondary Input

Servo Inputs

Summing Junction

Feedback Resistor

Follow-up Pot Arm

Center Tap

Follow-up Pot H₁

Follow-up Pot L₀

Input for voltmeter servo

Grounded for summing servo

All I Meg

IM Gain Control

Vibrator

IM   IM

Push-pull Amplifier

Damping Control

Critical Damping Switch

5K   .5μ   100K

Damping Tachometer

A.C. Line

Servo-motor

+100v

−100v

+300v

100K

Follow-up Potentiometer 20 K

47 K

−190v

Mechanically coupled to:

(a) A, B, C, and N pots and resolver on multiplier

(b) Drum on input table

(c) Pen or drum on output table

3. Resolving Servos

Vector resolution or summation is possible by utilization of the internal wiring (shown in Figure 15) of a group of three multiplying servos. In polar operation, shown in schematic form in Figure 16(a), x and y are fed in as inputs and $\tan^{-1} \frac{y}{x}$ and $\sqrt{x^2 + y^2}$ appear as outputs. In rectangular operation, x, y, and θ are the inputs with $y \sin \theta + x \cos \theta$ and $y \cos \theta - x \sin \theta$ as the outputs, as illustrated in Figure 16(b). Multiplications by x, y, and θ are possible simultaneously with the resolving action.

Table IV gives the results of various combinations of input signs and servo connections (summing or voltmeter). With summing servos, x, y, and θ may be the sum of several variables.

At present, the accuracy of $\tan^{-1} \frac{y}{x}$ computation is comparable with other machine operations if x and y are kept less than 60 volts in magnitude, but filter delay and drift of the 20 v a.c. power supply combine to make other operations unreliable.
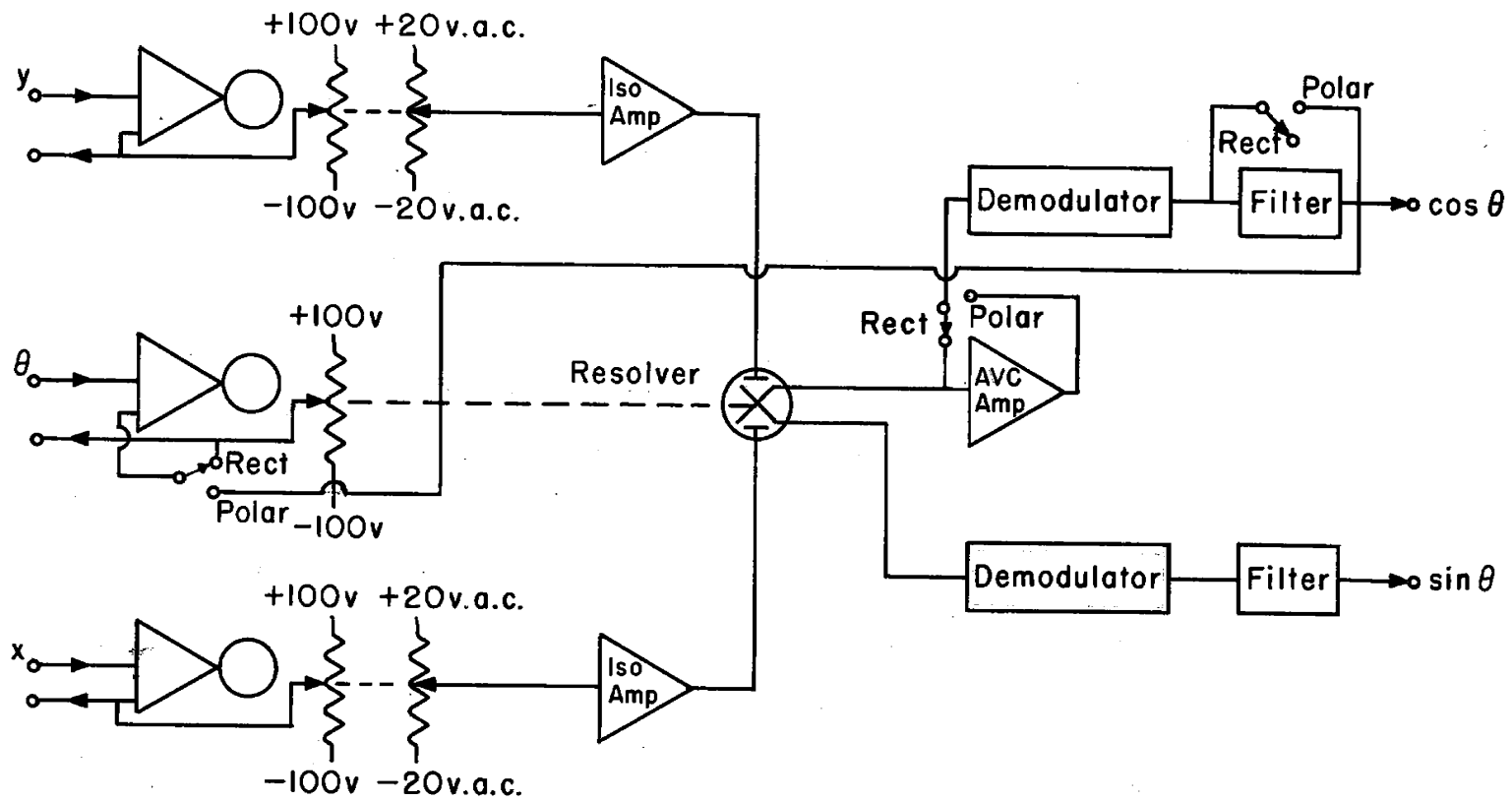
Fig. 15—Block diagram of resolving servo
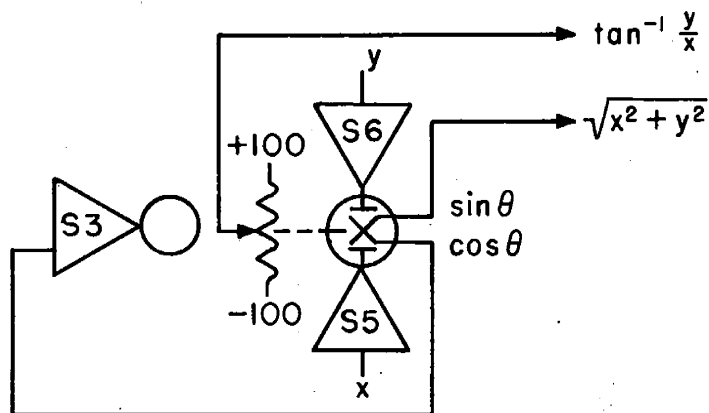
(a) Polar operation
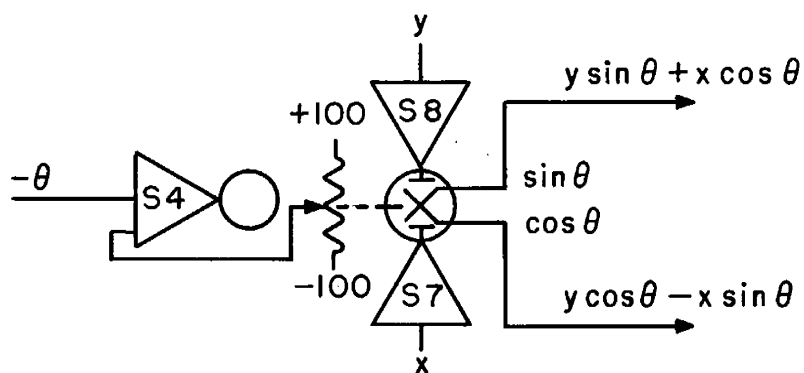
(b) Rectangular operation

Fig. 16 — Schematics of resolving servos

TABLE IV. RESOLVING SERVO COMBINATIONS

Function	Amplifier Number θ	x	y
Group I	3	5	6
II	4	7	8
III	11	13	14
IV	12	15	16

Computation	Amplifier Connection θ	x	y	Inputs θ*	x	y	Outputs θ*	sin θ	cos θ
Polar	Σ	Σ	Σ		+x	+y	+tan⁻¹ y/x	+ √(x²+y²)	
	Σ	V	V		-x	-y	+tan⁻¹ y/x	+ √(x²+y²)	
Rectangular	Σ	Σ	Σ	-θ	+x	0		+x cos θ	-x sin θ
	Σ	Σ	Σ	-θ	0	+y		+y sin θ	+y cos θ
	Σ	Σ	Σ	+θ	+x	0		+x cos θ	+x sin θ
	Σ	Σ	Σ	+θ	0	+y		-y sin θ	+y cos θ
	Σ	V	V	-θ	+x	0		-x cos θ	+x sin θ
	Σ	V	V	-θ	0	+y		-y sin θ	-y cos θ
	Σ	V	V	+θ	+x	0		-x cos θ	-x sin θ
	Σ	V	V	+θ	0	+y		+y sin θ	-y cos θ
	V	Σ	Σ	-θ	+x	0		+x cos θ	+x sin θ
	V	Σ	Σ	-θ	0	+y		-y sin θ	+y cos θ
	V	Σ	Σ	+θ	+x	0		+x cos θ	-x sin θ
	V	Σ	Σ	+θ	0	+y		+y sin θ	+y cos θ
	V	V	V	-θ	+x	0		-x cos θ	-x sin θ
	V	V	V	-θ	0	+y		+y sin θ	-y cos θ
	V	V	V	+θ	+x	0		-x cos θ	+x sin θ
	V	V	V	+θ	0	+y		-y sin θ	-y cos θ

*Scale Factor of 1/2 volt per degree

D. Input Tables

Figures 17 and 18 illustrate one of the five large input tables equipped for automatic tracking. A typical grouping of two smaller input tables mounted on a servo rack is shown in Figure 19. The schematic notation for an input table is given in Figure 20.

Both types of input tables operate on the same principle. A wire is cemented over a graph of the curve to be tracked and the graph paper placed on the drum with the independent variable or argument in the direction of drum rotation. The drum is rotated by the independent variable in such a manner as to keep the linear slide wire (mounted parallel to the drum axis) over the correct value of the independent variable on the graph paper at all times. The wire on the curve then acts as the arm of a potentiometer tapping off the proper fraction of the voltage across the slide wire as dictated by the curve. Thus multiplication is possible simultaneously with the introduction of an arbitrary function if the multiplier voltage is applied across the slide wire.

The function to be tracked is plotted point by point on 11 x 19 inch graph paper (K and E No. 358-11L) for the large tables and on 8-1/2 x 11 inch graph paper (K and E No. 358-11) for the servo-rack input tables. A piece of about 0.015 inch diameter wire is first straightened by stretching it slightly beyond its yield point. It is then formed to pass through the plotted points of the function and is held in position by small double needle point clamps (Figure 21) or by strips of masking tape. This operation is often simpler than the usual French curve fitting that is necessary when the function is to be drawn for manual tracking. For high accuracy work the loading of the slide wire by the input of the amplifier it feeds into should be compensated for by proper shifting of the plotted points or by an unloading amplifier. Next the wire is cemented to the graph paper using a solution of about five parts of General Cement Company No. 30-8 Radio Service Cement
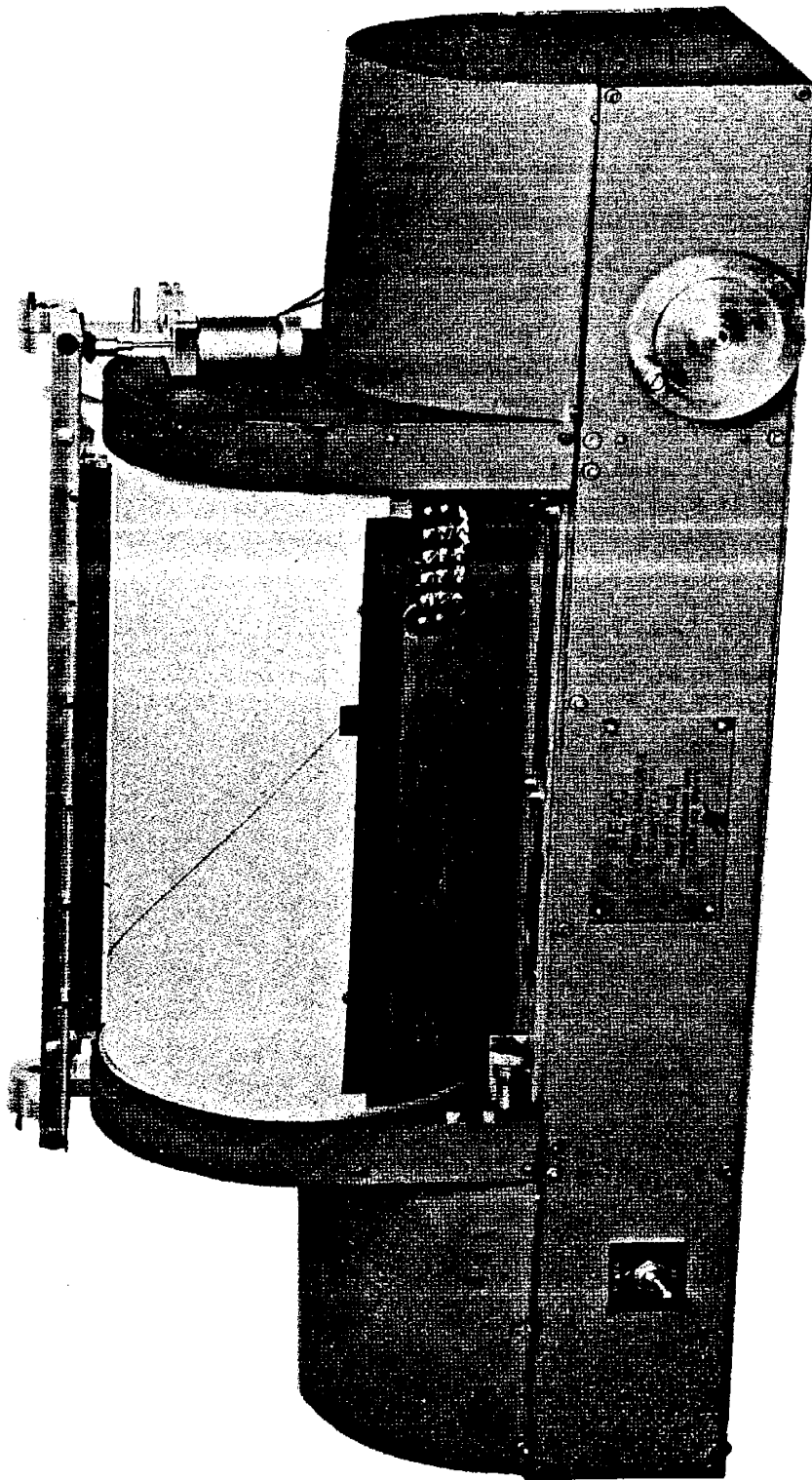
Fig. 17 — Input table — slide wire down
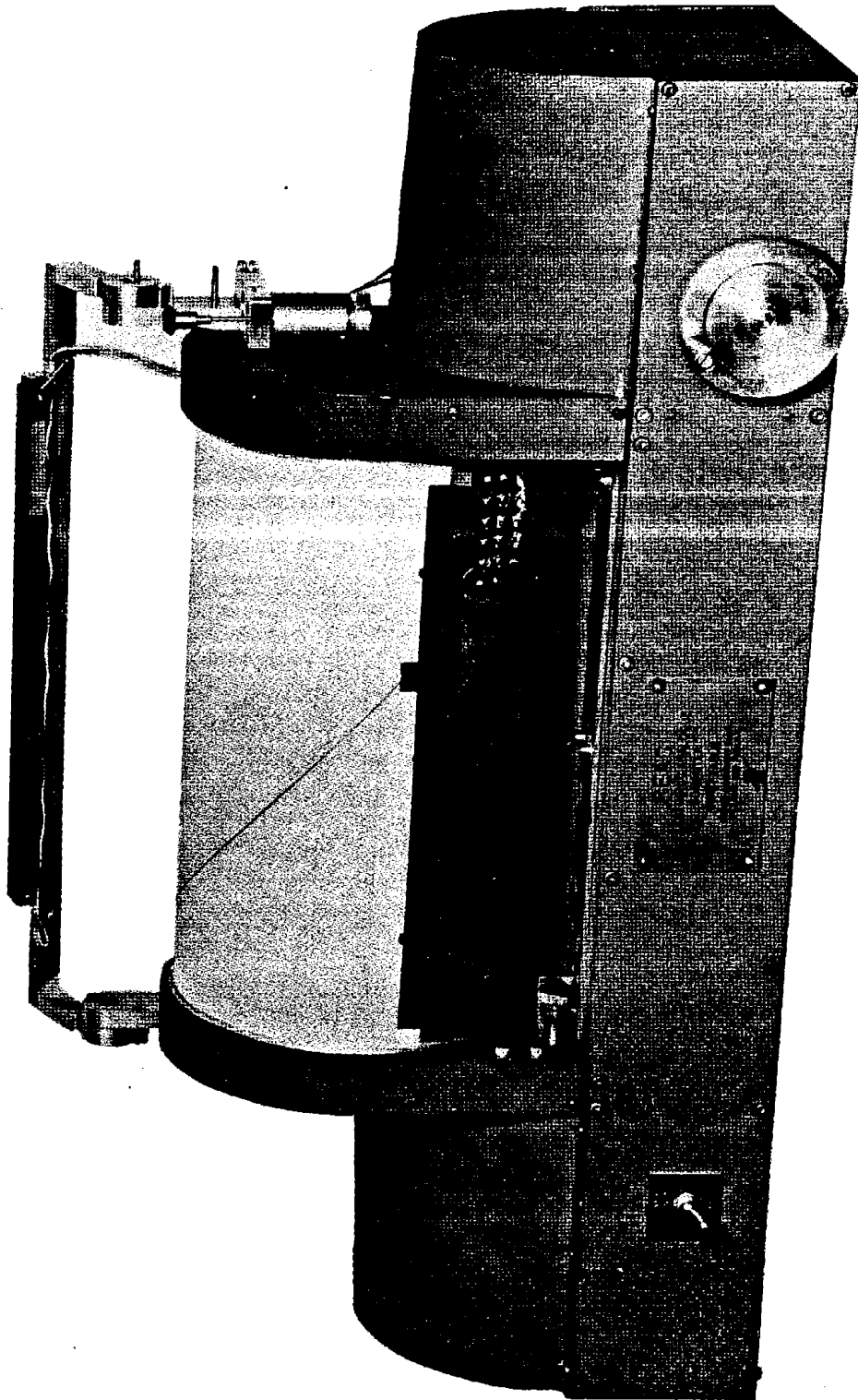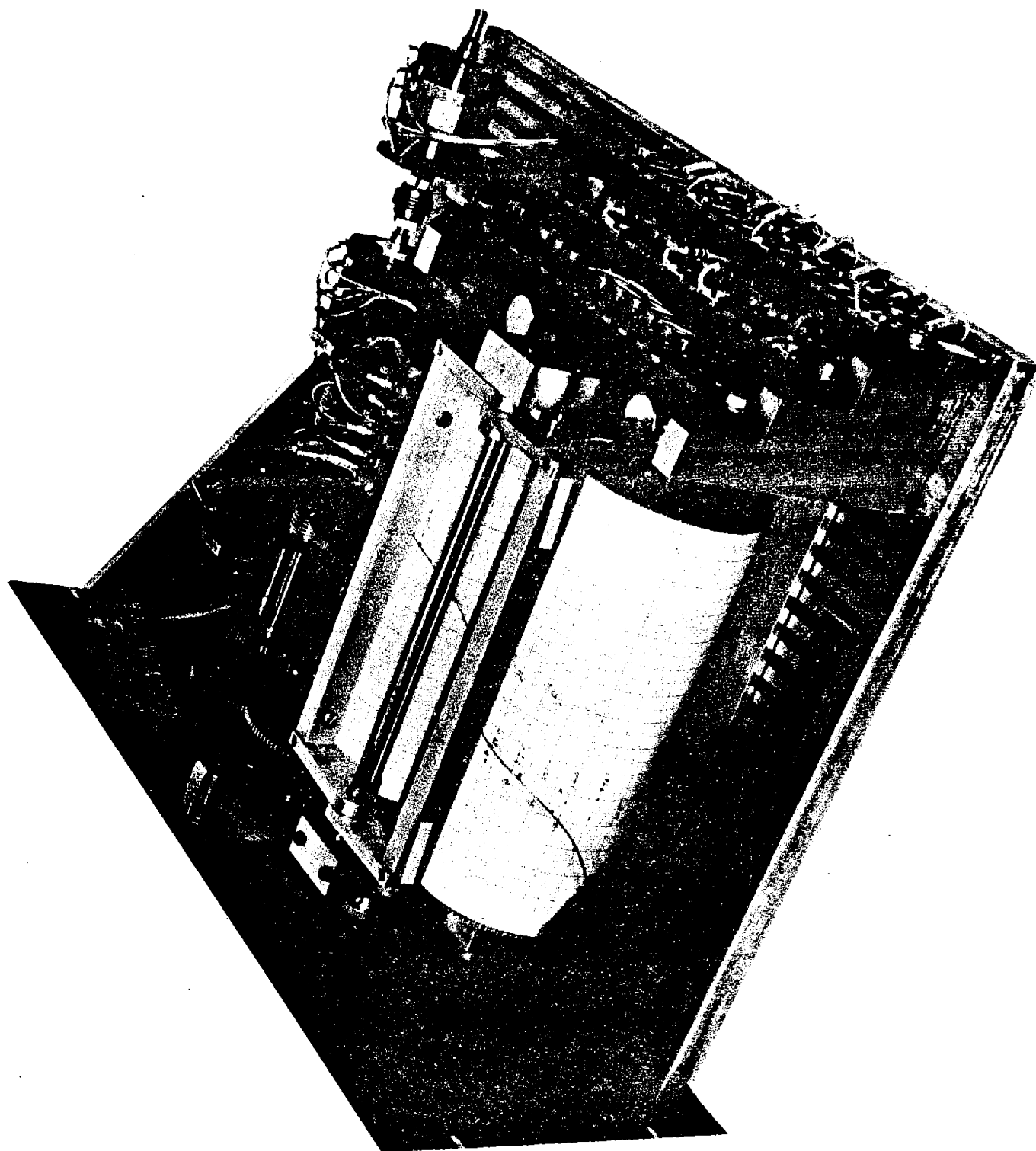
Fig. 19— Input table mounted on servo rack
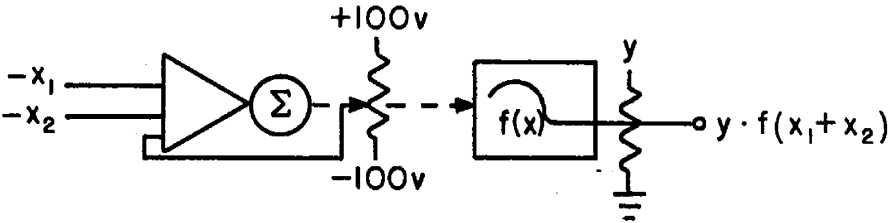
Fig. 20—Schematic diagram of an input table

Fig. 21 —Preparation of arbitrary function for automatic tracking

to one part of No. 31-8 Radio Service Solvent. After a few minutes the clamps or tape strips are removed to allow the rest of the wire to be cemented. The top of the wire is cleaned after another minute or two drying time by wiping with crocus cloth or solvent to assure good contact with the slide wire. The whole preparation time takes less than fifteen minutes per curve.

The linear slide wires are 20,000 ohms plus or minus one per cent total resistance, ten inches +0.02, -0 inches in length with a ± 0.1 per cent linearity tolerance.

Notice that it is possible to have more than one wired curve on a given input table if intersection of the functions can be avoided. Obviously the functions must have the same independent variable. This procedure permits automatic tracking of a function of two independent variables by means of linear interpolation between parametric plots. When several wires are present on the same graph paper it is necessary to use a thin rubber cushion over the input table drum to insure reliable contact of all function wires with the linear slide wire (Figures 17 and 18). Satisfactory contact is obtained with a net downward force of 3 oz. per wired curve at the slide wire, although this value depends somewhat on the thickness of the rubber cushion used. It is necessary to compensate for the change in the effective drum circumference by either reducing the size of the feedback resistor of the servo motor or reducing the independent variable voltages by potentiometers.

It is planned as a future project to put low resistance current indicators in series with each slide wire; the absence of current flow with voltage across the slide wire will indicate an opened slide wire. Should the Markite conducting plastic "slide wires" prove satisfactory, this will not be necessary.

E.   Output Tables

Nearly all of the REAC recording has been done on the input-output table shown in Figure 22. One of the large input tables can be used as an output table when needed and three Esterline-Angus recorders (Figure 23) are available for plotting secondary variables as functions of time.

Figure 22 shows the pen lifting device normally used to permit recording only when the operate button has been pushed. However, the pen lifter can be controlled externally to perform such "tricks" as lifting at curve discontinuities, giving pips at equal intervals of time, etc. Two timing devices (6 and 12 rpm) for the latter application have been added as RAND REAC components.

Figure 24 is the schematic notation for an output table.

When curves are to be reproduced, vellum graph paper (K and E No. 359T-11LG) and the Leroy pen with black India ink should be used.
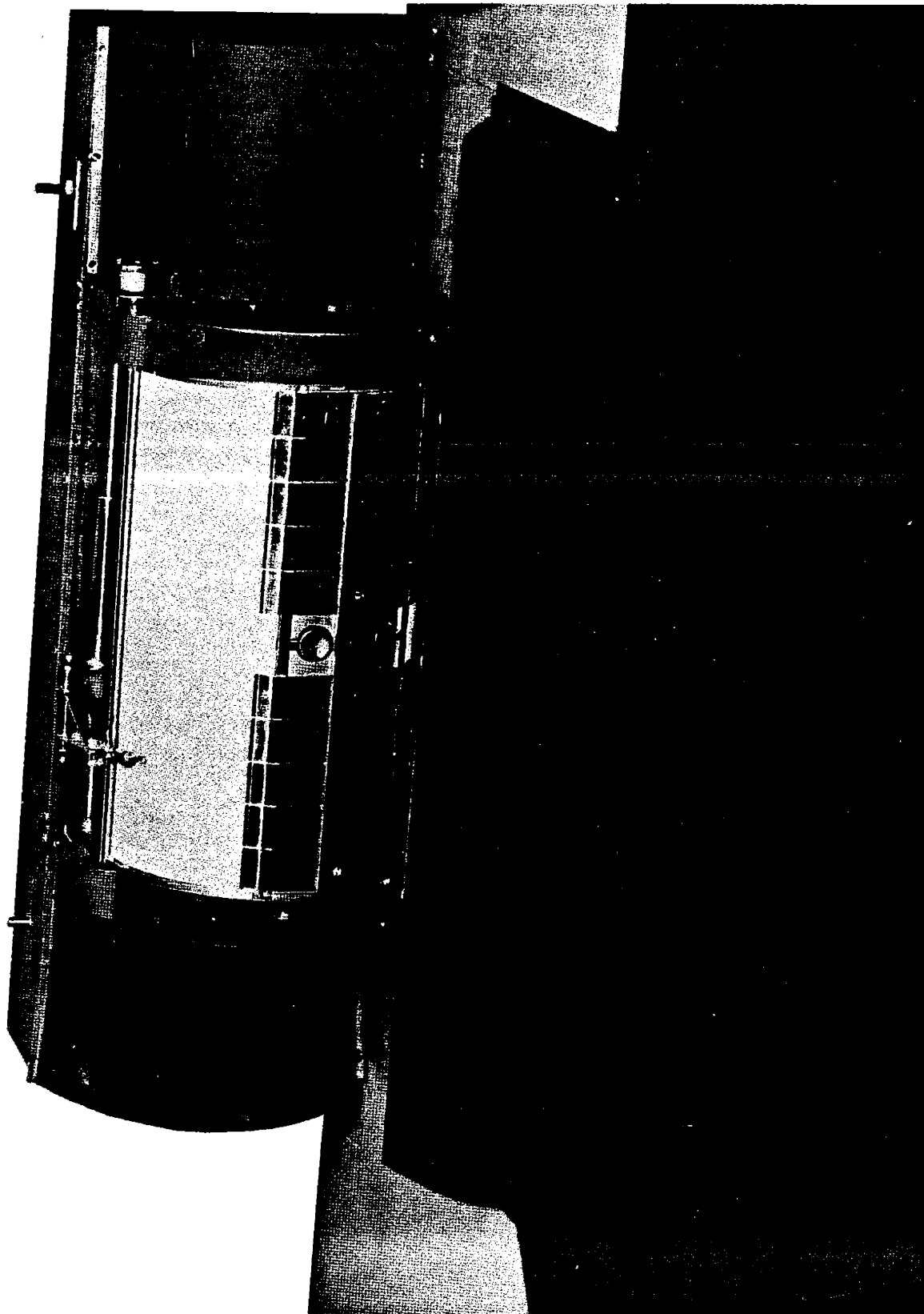
Fig. 22 — Input — output table
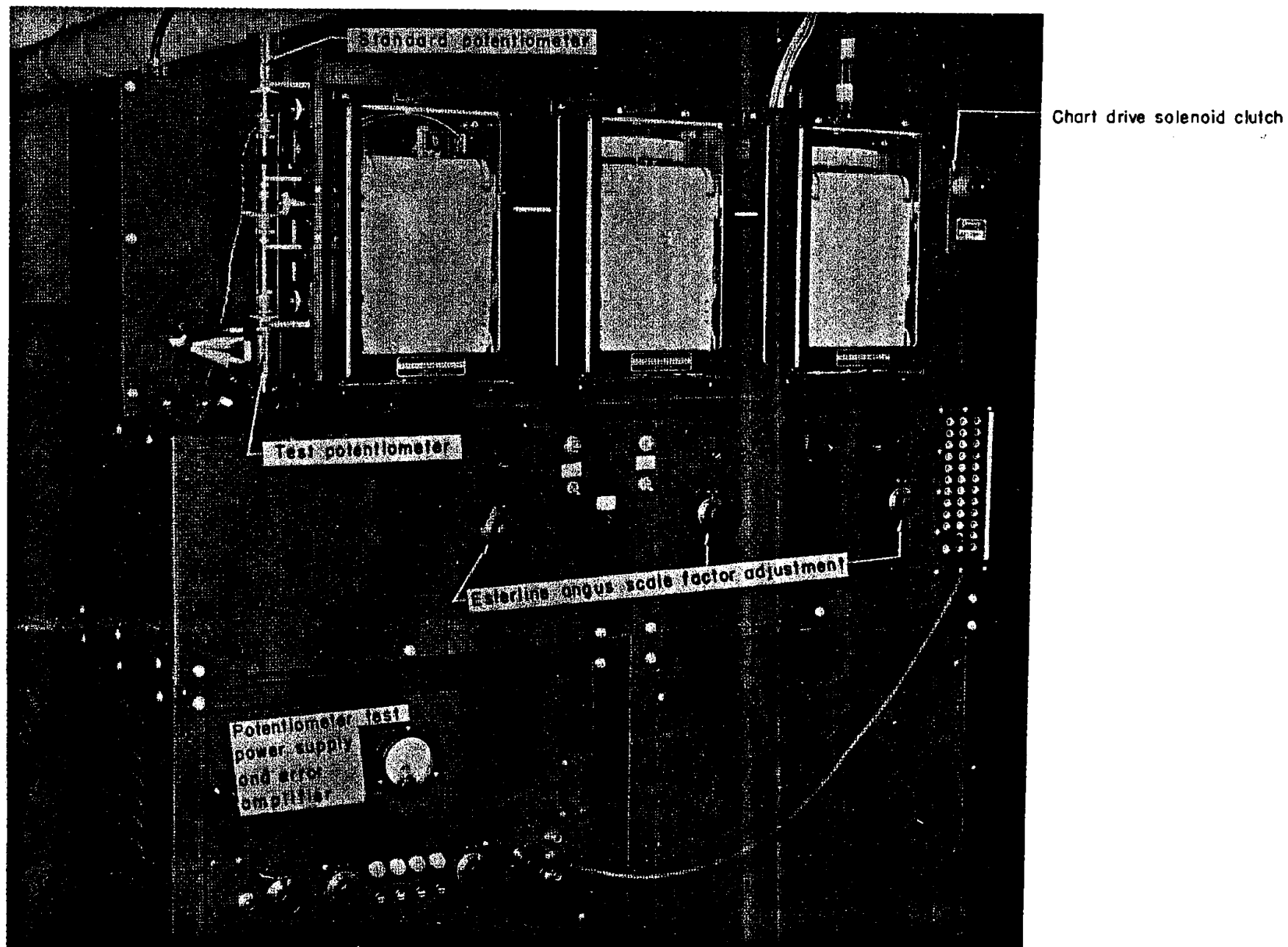
Fig.23—Esterline angus output recorders and potentiometer test fixture
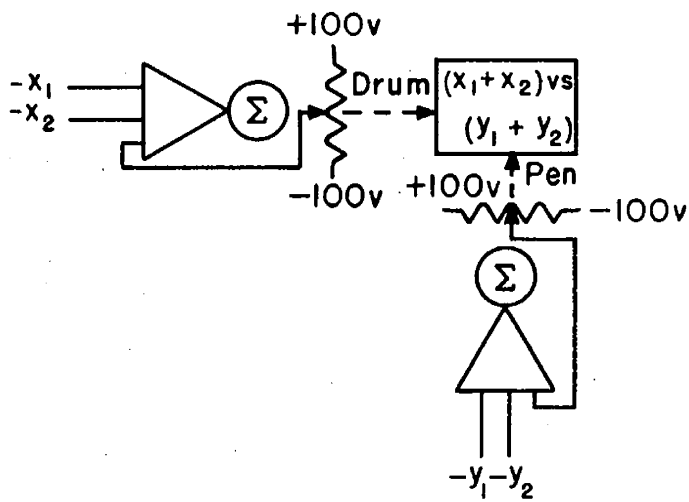
Fig. 24——Schematic diagram of an output table

F. Relays

In addition to the relays connected to each amplifier, several external ones, shown schematically in Figure 25, have been added to facilitate various automatic switching schemes. With no voltage across the coil the A arms are connected to the NC (normally closed) contacts, but when a voltage of sufficient size is placed across the coil all three A arms change over to make contact with the NO (normally open) contacts. The relays take approximately ten milliseconds to switch position. The resistance in series with the coil has been added to permit driving the coil with a high gain amplifier. Six SPDT polarized relays also have been added.

G. Limiters

Sometimes it is necessary to bound a variable between two limits; the REAC has four limiters to perform this task. The circuitry of a limiter is given in Figure 26 and the schematic notation in Figure 27. The need for a dropping resistor demands that one-fourth the limits be introduced and that the output of the limiter goes into the subsequent amplifier at a gain of four.

H. Control Switches

Push buttons are provided for initiating solutions (operate), stopping solutions (hold), and returning to the original starting values (reset). These and other controls and their functions will be described more completely in the section on machine operation.
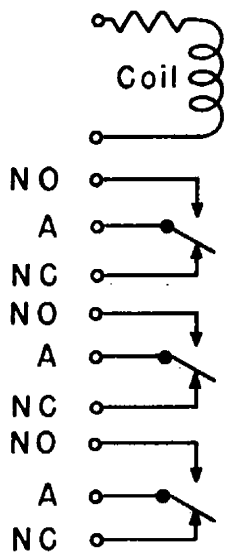
Fig. 25—Relay circuit schematic diagram
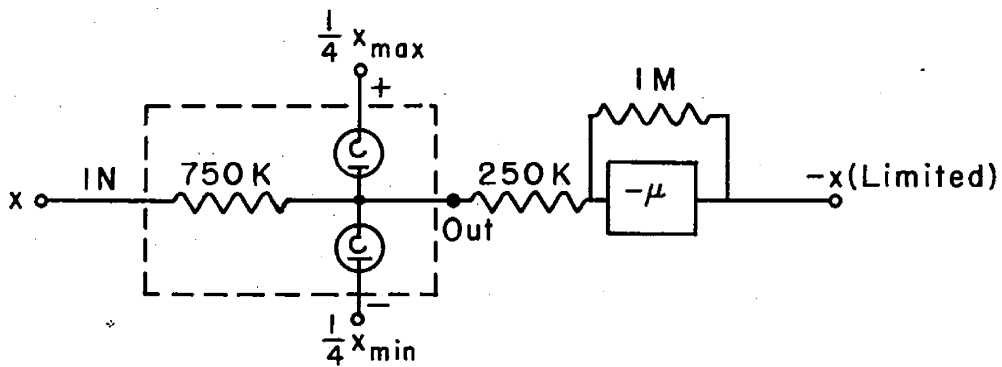
Fig. 26—Limiter circuit
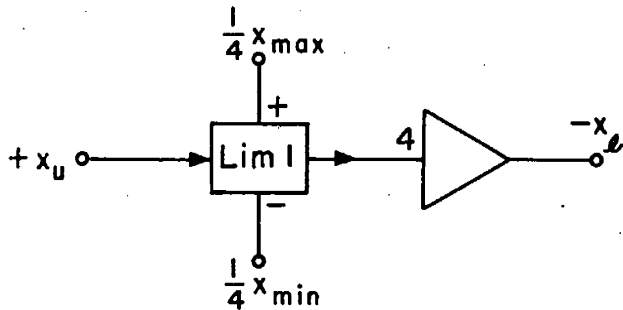
Fig. 27—Limiter schematic diagram

I. Summary of RAND Modifications of the REAC

The RAND modifications of the REAC have had as a
purpose the improvements of the accuracy and flexibility of a
machine designed more for guided missile simulation than for
the demanding problems met at RAND.

Since the accuracy of a computing machine in general
can be no better than that of its least accurate element, great
care has been taken to select, modify, and trim components to
meet the following tolerances:

a) The magnitudes of the plus and minus 100 volt
power supplies within ±0.001 volt of each other.

b) The gains of summing, integrating, and servo
amplifiers accurate within ±0.01 per cent.

c) Constant factor potentiometer settings within
±0.1 per cent of their readings.

d) Servo dial reading maximum error ±0.1 volt.

e) Servo multiplier products inaccuracy less than
0.1 per cent of the variable across the multiplying
potentiometer.

f) Input curve maximum error of 0.1 per cent of full
scale.

The most important RAND modification has been the
replacement of the telephone type jack panel with the removable
IBM-type plugboard. Not only have the many hours of machine time
wasted while a problem is being patched-in been eliminated, but
once a plugboard has been found to work satisfactorily on a problem,
subsequent reruns may be made with but a few minutes notice. Also,
check boards may be used to test the operation of all components and

rapidly put the finger on faltering equipment. Moreover, the
operating console has been so designed as to greatly simplify the
life of the operator. Most controls are push-buttons; dials, controls,
output table, etc., are within easy reach; and a digital readout
has been added. Figures 28 and 29 show the "before and after"
views of the operating controls and patch board. AN IBM card punch
has been connected to the digital readout to permit error-free
recording of initial settings and final results.

Contacts on the plugboard to the grid and summing
junctions of all amplifiers have been a major factor in increasing
the flexibility of the REAC. Any type of impedance may be used as
an input or feedback element, although those for summing, integrating,
or high gain amplifiers are employed most often. The REAC as
received had seven integrating, thirteen summing, and three
differentiating amplifiers and as such could not have done many
problems solved on the modified version in which all amplifiers
are essentially identical. For example, one problem required
thirteen integrating amplifiers and only five summing amplifiers,
while on the other extreme another problem required twenty summing
amplifiers and only three integrating amplifiers.

Extra banks of condensers and resistors have been added
to permit variation in input and feedback impedances, and all input
and feedback resistors and condensers have been mounted in individual
cabinets to minimize the temperature and humidity differential.
All amplifiers have six unity gain inputs; other values of input
gain require selection of the proper impedances from the spare
impedance bank.

Plugboard control of amplifier relays is necessary to
permit summing or integrating with each amplifier. This access to
the hold relays was found to be so useful that twelve other relays
controlled from the plugboard have been added.

Modification of the multiplying and input and output
table servo amplifiers to permit summing has greatly augmented
the number of summing amplifiers. Moreover, multiplying
potentiometers have been geared to all modulator servos, doubling
the number of multiplying servos. The feedback loops on the servo
amplifiers may be opened and the follow-up potentiometers may be
connected as desired from the plugboard. The use of tapped poten-
tiometers not only increases the accuracy of multiplication, but
often reduces the need of inverting amplifiers, permits the
automatic tracking of a function of two variables, and makes
excellent limiters out of the servos. Eleven-turn tapped poten-
tiometers permit the servos to run between $\pm 100V$ instead of $\pm 98V$
as in the original equipment.

The input tables have been modified from hand tracking
to automatic by the use of the wired curves discussed above. The
resultant accuracy and precision is better than that attainable
with human trackers and the operating time depends only upon the
frequency response of the drum. The savings resulting from the
elimination of hand trackers has averaged over $1000 per month;
while the modification cost was less than $50.00 per table.

Selector switches have been added to bring to a common
point the output of any amplifier, constant factor potentiometer,
or servo potentiometer. A push button replaces the inputs of all
constant factor potentiometers with +100 volts to permit digital
readout setting of the potentiometers under their actual loading
conditions.

Multithrow function switches have been provided for
convenience in modifying circuits, selecting outputs for plotting,
etc.

III. PROBLEM PLANNING

A. Basic Planning

Basic planning techniques can be described best by discussing examples of increasing complexity.

Example 1

One of the simplest differential equations

$$\frac{dx}{dt} = \dot{x} = k$$

$$x_0 = x(t=0) = A$$

is also one of the most useful. Since its solution is

$$x = kt + A$$

this differential equation is the basis of the generation of a voltage proportional to time. If a constant voltage k is fed into an integrator with an initial setting of A, the output will be (kt + A).

In the REAC all voltages usually lie between ± 100 volts, and several ± 100 volt outlets are available on the plugboard. If -100 volts is introduced into an integrator having an input impedance of ten megohms, a feedback condenser of ten microfarads, and an I.C. of zero, the output will be $100 \cdot \frac{1}{10} \cdot \frac{1}{10} t = t$ volts, as shown in Figure 30.

Example 2

Another equation that appears quite often is

$$\dot{x} = -bx$$

$$x_0 = A$$

We start off assuming we have $\dot{x}$ available and operate upon it until we obtain the necessary ingredients of $\dot{x}$. As shown in Figure 31, integration of $\dot{x}$ yields $-x$ which when multiplied by b with a
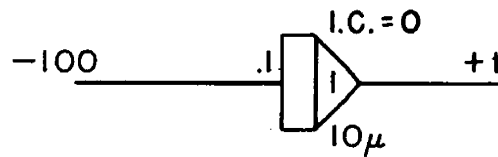
Fig. 30—Generation of a voltage proportional to time
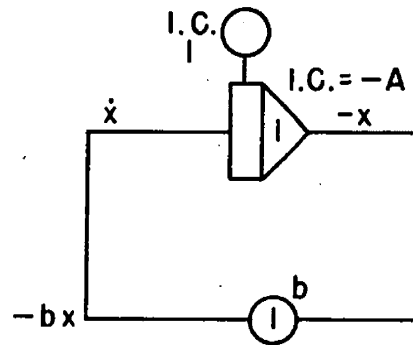
Fig. 31—— Circuit for solving $\dot{x}=-bx$
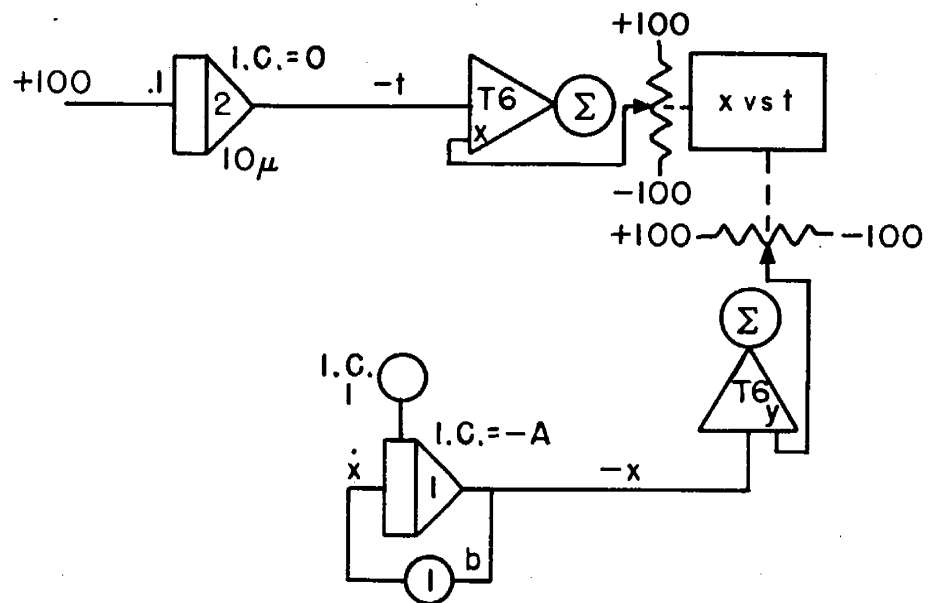
Fig. 32——Circuit for plotting x vs t

constant factor potentiometer is equal to $\dot{x}$ and hence becomes
the input to the integrator. The I.C. of the integrator is set
to give an output of -A volts, since the output is -x. If a plot
of x vs t is required, the circuit becomes that of Figure 32.

Example 3

A typical homogeneous, linear differential equation with
constant coefficients might appear as

$$\ddot{x} + a\dot{x} + bx = 0$$
$$\dot{x}_0 = 0$$
$$x_0 = A$$

Solving for the highest order derivative we obtain

$$\ddot{x} = -a\dot{x} - bx$$

As before, we assume the highest order derivative, $\ddot{x}$, is available
and we operate upon it until the terms in its equation are found.
The schematic diagram for its solution is given in Figure 33.
Notice that the sole purpose of amplifier No. 9 is to change the
sign of x.

If the equation is modified to the complete form

$$\ddot{x} + a\dot{x} + bx = f(t)$$

and a plot of x versus t is required, the schematic becomes that
of Figure 34.

Example 4

As an example of the solution of a set of simultaneous
differential equations, consider the following system:

$$\dot{x} + ax + by = c$$
$$\dot{y} + dx + ey = f$$
$$x_0 = A$$
$$y_0 = B$$

Fig. 33—Circuit for solution of $\ddot{x} + a\dot{x} + bx = 0$

Fig. 34—Circuit for solution of $\ddot{x} + a\dot{x} + bx = f(t)$

The customary analytic approach would be to reduce the system to a single characteristic second-order equation. The resulting computation of coefficients and initial conditions is eliminated in the REAC solution, which uses the equations in the form

$$\dot{x} = c - ax - by$$
$$\dot{y} = f - dx - ey$$

Again it is assumed that $\dot{x}$ and $\dot{y}$ are available and are operated upon to form the terms of the right side of the equations, as shown in Figure 35.

Example 5

As an example of a servo-multiplier application consider the generation of the power series

$$x = a + bt + ct^2 + dt^3 + et^4$$

A schematic for this equation is shown in Figure 36. Particular notice should be given to the sign of the variables on the potentiometer arms and the use of isolating amplifiers.

This particular equation could be solved with much less equipment by solving the equivalent differential equation

$$\ddddot{x} = (4!)e, \text{ with}$$
$$\dddot{x}_0 = (3!)d$$
$$\ddot{x}_0 = (2!)c$$
$$\dot{x}_0 = b$$
$$x_0 = a.$$

Example 6

The following non-linear differential equation illustrates the combination of amplifiers and servo-multipliers in one circuit:

$$\ddot{y} + 2\alpha y\dot{y} + (wy)^2 = f(y)$$
$$\dot{y}_0 = 0; \; y_0 = A$$
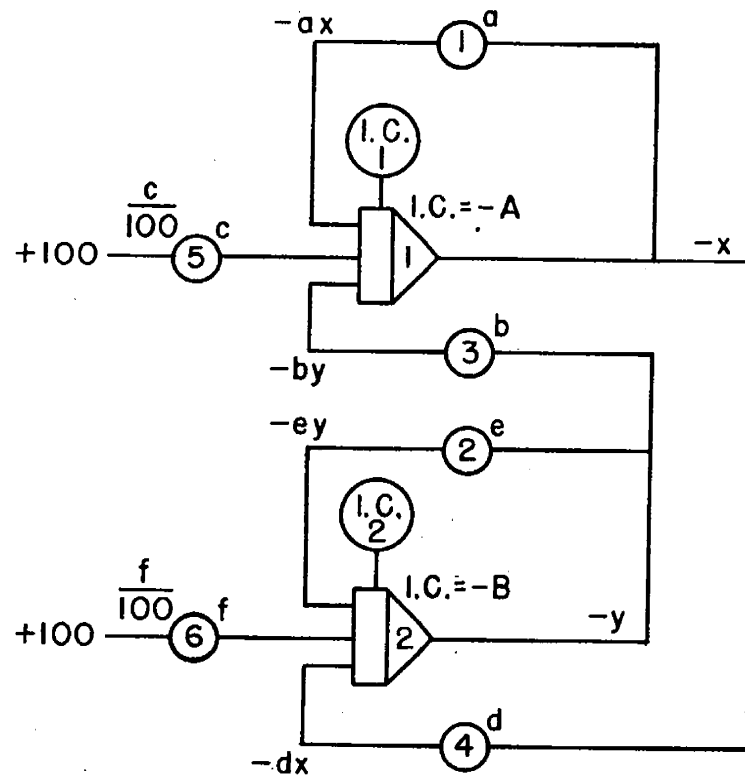
Fig. 35—Circuit for solution of

$$\dot{x} + ax + by = c$$
$$\dot{y} + dx + ey = f$$
$$x_0 = A$$
$$y_0 = B$$
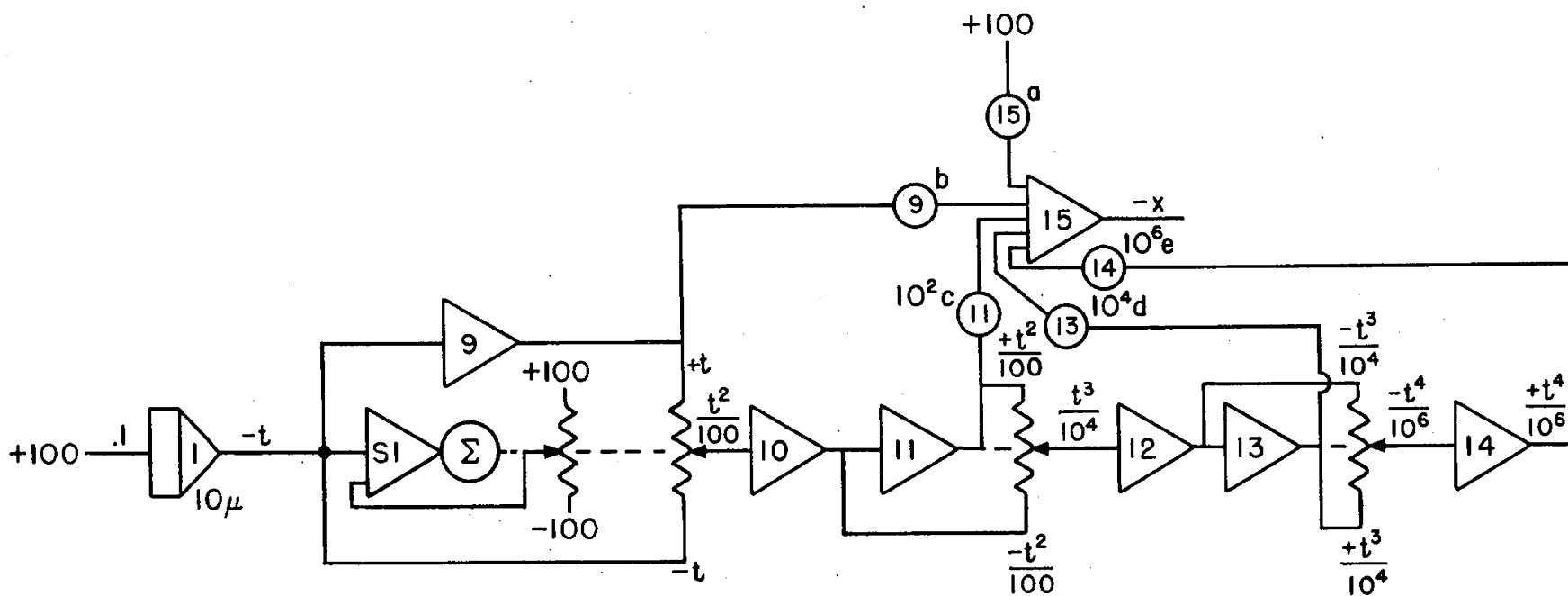
Fig. 36—Schematic for generation of power series

$$x = a + bt + ct^2 + dt^3 + et^4$$

The schematic for this equation, shown in Figure 37, illustrates
the previous statement that the REAC is not upset by a non-linear
equation, despite the analytic difficulties inherent in such cases.
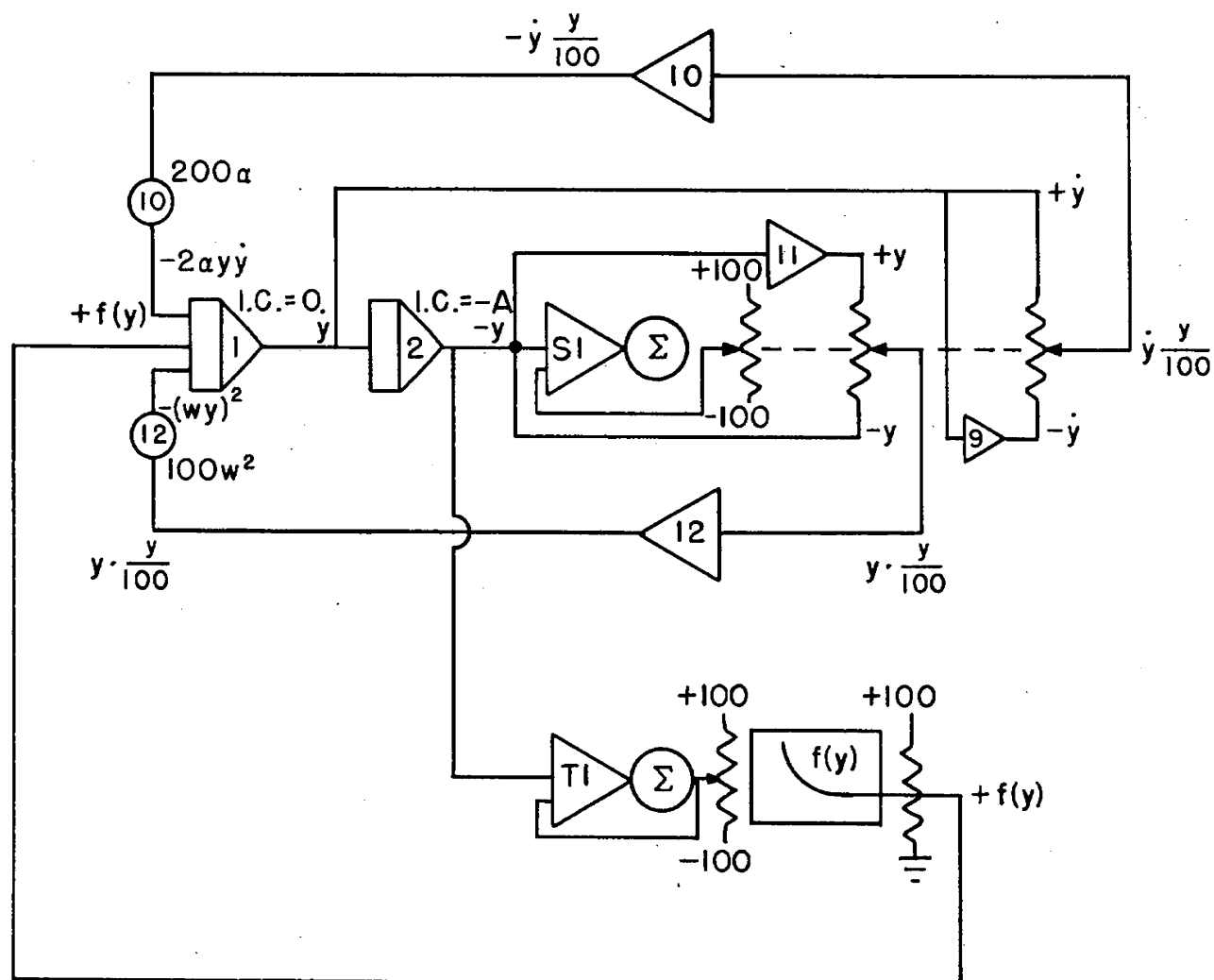
Fig. 37—Circuit for solution of $\ddot{y} + 2a y \dot{y} + (wy)^2 = f(y)$

B. Change of Variable

If integrations are considered to be of the form

$$\frac{d^n x}{dt^n} = \int \frac{d^{n+1} x}{dt^{n+1}} \cdot \frac{dt}{d\tau} \cdot d\tau$$

and

$$t = \int \frac{dt}{d\tau} \cdot d\tau$$

certain flexibility of operations results by various choices of $\frac{dt}{d\tau}$. Notice that τ is the machine time scale variable while t is the independent variable of the problem. For example, all derivatives can be put across servo multiplying potentiometers before going to the integrators and the servo positioned by a hand-set potentiometer, permitting operator selection of the computing speed. This technique is useful for slowing the computing rate at crucial periods when hand-tracking or stopping the solution by hand. If both signs of the derivative are put across the potentiometers the solution may be stopped, or even reversed, by proper setting of the input potentiometer. Figure 38 illustrates a typical schematic.

Integration with respect to a variable other than the independent variable is similarly achieved, i.e.,

$$\frac{d^n y}{dx^n} = \int \frac{d^{n+1} y}{dx^{n+1}} \cdot \frac{dx}{d\tau} \, d\tau = \int \frac{d^{n+1} y}{dx^{n+1}} \cdot \frac{dx}{dt} \frac{dt}{d\tau} \, d\tau.$$

The following problem illustrates automatic time scale adjustment and integration with respect to a variable other than the independent variable.

It was desired to find P as a function of x, such that $.20 \leq P \leq .99$ where
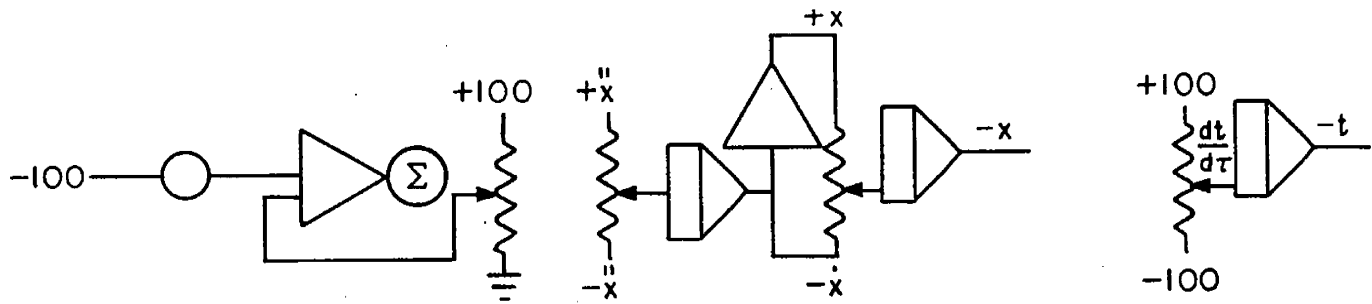
Fig. 38—Manual control of computing speed
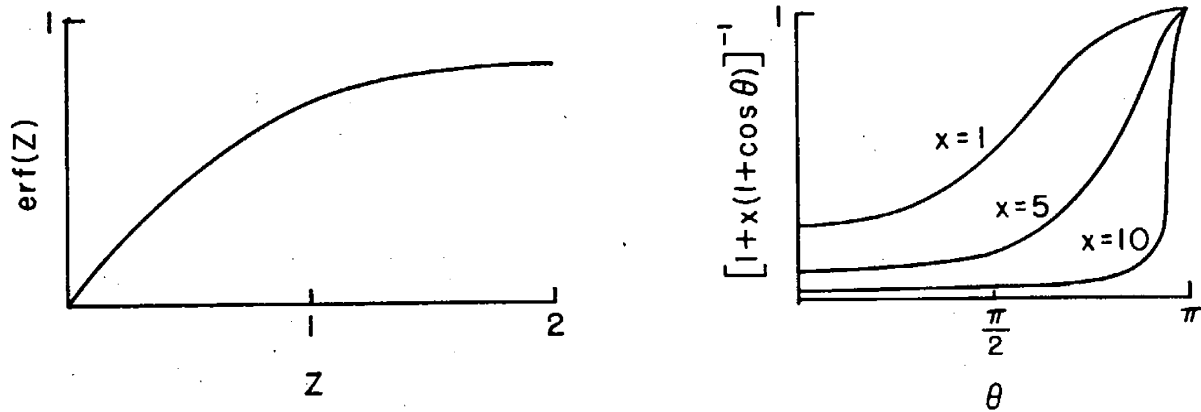
Fig. 39—Function values for example

$$P(x) = 1 - \frac{1}{\pi} \int_0^\pi \text{erf}\left(\frac{3.412}{\sqrt{1+x+x\cos\theta}} \right) d\theta$$

in which
$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt.$$

It turns out that large values of x are necessary to give the values of $P(x)$ of interest. The argument of the error function and thus the integral are such that the major contribution to $P(x)$ occurs for a small range of θ near π (Figure 39). Therefore, let $\theta = \pi(1 - e^{-\tau})$ in order to spread out the integrand. Thus,

$$P(x) = 1 - \int_0^\infty e^{-\tau} \text{erf}\left[\frac{3.412}{\sqrt{1+2x\left(\frac{1+\cos\theta}{2}\right)}} \right] d\tau = 1 - \int_0^\infty e^{-\tau} \text{erf}\left(\frac{3.412}{\sqrt{1+y^2}} \right) d\tau$$

where $y = \sqrt{2x} \cos \frac{\theta}{2}$. Here $w = e^{-\tau}$ is obtained as a solution of $\frac{dw}{d\tau} = -w$ and $y = \sqrt{2x} \cos \frac{\theta}{2}$ is a solution of $\frac{d^2y}{d\theta^2} = -\frac{1}{4}y$.

However, since the REAC will integrate only with respect to time, y must be obtained as a solution of $\frac{dy}{d\tau} = \frac{dy}{d\theta} \cdot \frac{d\theta}{d\tau} = \frac{dy}{d\theta}(+\pi w)$. The term $\frac{dy}{d\theta}$ may be obtained by solving $\frac{d}{d\tau} \frac{dy}{d\theta} = \frac{d^2y}{d\theta^2} \frac{d\theta}{d\tau} = -\frac{\pi}{4} yw$. The function erf $\frac{3.412}{\sqrt{1+y^2}}$ was plotted against y and tracked from an input table. The function w, which was obtained from the REAC, was applied to the input table slide wire. The product was then integrated with respect to time; this quantity subtracted from 1 then gave the value of P corresponding to the value of x set as an initial condition in the solution for y.

Changes of variables that simplify computations or
improve the accuracy are often possible. The following problems
illustrate such techniques. Certain scale factor difficulties in the
van der Pol equation $\frac{d^2x}{dr^2} + \mu(x^2 - 1)\frac{dx}{dr} + x = 0$ could be avoided
by considering the equivalent equation $\frac{d^2x}{dt^2} + (x^2 - 1)\frac{dx}{dt} + \varepsilon^2 x = 0$
where $t = \mu r$ and $\varepsilon = \frac{1}{\mu}$. As μ is increased to make the van der Pol
equation highly non-linear, the voltage representing the derivative
increases rapidly as the sides of the wave form become more nearly
vertical. In the transformed equation this increase in voltage is
partially off-set by expanding the time scale as μ increases.
Solutions to this equation were plotted in the (x,t) plane and in
the $\left(x, \frac{dx}{dt}\right)$ plane for $\varepsilon = .001$ to 100. Rayleigh's equation,
$\frac{d^2z}{dr^2} + \mu\left(\frac{1}{3}x^3 - x\right) + z = 0$, where $x = \frac{dz}{dr}$, is equivalent to the system
$\frac{dx}{dt} = -\left(\frac{1}{3}x^3 - x\right) - y$, $\frac{dy}{dt} = \varepsilon^2 x$ where $y = \varepsilon z$, $t = \mu r$, and $\varepsilon = \frac{1}{\mu}$.
A plot was made of the phase portrait in the (x,y) plane for the
same values of ε as above.

The following problem resulted from a study of the
laminar boundary layer in gases. The simultaneous differential
equations to be solved were

$$g \cdot \frac{d^2g}{dx^2} + xA = 0$$

$$\frac{dg}{dx} \cdot \frac{dj}{dx} = \frac{d}{dx}\left(\frac{1}{P} \cdot \frac{dj}{dx} \cdot g\right) + g$$

where A and P were both plotted functions of j which were entered
by means of input tables. It was desired to obtain end values of

j in the interval $1 \leq j \leq 4$, the end value being that obtained
when $g = 0$. In this form, g and j approach final values with
steep slopes since $\frac{d^2g}{dx^2} \rightarrow \infty$ as $g \rightarrow 0$. With the transformation,
$g = \frac{dx}{dt}$, the equations were reduced to:

$$\frac{dg}{dt} = g \cdot \frac{dg}{dx} \; ,$$

$$\frac{d}{dt}\left(\frac{dg}{dx}\right) = -xA \; ,$$

$$\frac{dj}{dt} \cdot \frac{dg}{dx} = \frac{d}{dt}\left(\frac{1}{P} \frac{dj}{dt}\right) + g^2$$

In the transformed equations g and j approach the final values
asymptotically.

C. Scale Factor

Although the variables of a problem to be solved on
the REAC can have any conceivable units, when placed on the
computer the variables must be expressed in volts. Hence, some
scale factor is necessary to translate each variable from its
natural unit to volts and back again. This scale factor is the
number of volts representing one unit of the variable and S_i is
the symbol for the scale factor of x_i.

Choice of scale factors usually involves compromises.
Large voltages (and as a consequence large scale factors) are
desirable to minimize the effect of stray voltages, to minimize
the relative inaccuracies of servo applications (which have a
positioning error of ± 0.1 volt irrespective of the magnitude),
etc. On the other hand, the voltages must be kept within the
± 100 volt range and when the bounds of a variable are not known,
the tendency is to use a small scale factor to assure proper
operation. Scale factors also influence the time required for
a solution and while a short computing time is desirable, the
relative delays of the servos must be kept small to minimize errors.

Scale factors are assigned to the driving voltages (I.C.,
input table, constants) and become modified in the computing network
by amplifier input gains, scale factor potentiometers, servo
multiplication or division, and redefinition of the function.
This latter technique permits use of the convenient scale factors
1 volt per unit or 100 volts per unit. For example, if x lies between
± 5 feet, the redefined variable 20x could have a scale factor of
1 volt per foot.

It is apparent that estimates of the bounds of all
variables are required. Since the estimates will not always be
correct the circuits must allow for pre-planned changes in scale-
factors. This anticipation of possible modifications is by no

means simple, but marks the difference between a well and poorly planned problem.

The basic scale factor rules:

1) Voltages to be summed by any d.c. or servo amplifier must have the same scale factor after multiplication by their respective input gains.

2) All servo operations have a setting of $\frac{x_1}{x_2}$ and yield $\frac{x_1 x_3}{x_2}$ at the coupled potentiometer arm, where

x_1 = input voltage

x_2 = voltage across follow-up potentiometer

x_3 = voltage across coupled potentiometer.

For most multiplication and input or output table applications x_2 is 100 volts, in which case the setting is $x_1/100$ and the output of a multiplying circuit is $x_4 = x_1 \cdot x_3$ with $S_4 = \frac{S_1 \cdot S_3}{100}$ or $\overline{x}_4 = x_1 \cdot \frac{x_3}{100}$ with $S_4 = S_1 \cdot S_3$.

3) Dependent variables on input tables should be normalized to f/f_{max}.

The handling of the independent variable is simplified by using the technique of the previous section. If t is the independent variable of the problem and T the machine time scale, we have

$$t = \int \frac{dt}{dT} \, dT.$$

For most problems

$$\frac{dt}{dT} = K = \frac{t_{max} - t_{min}}{\text{Solution time}}.$$

In this case,

$$\frac{d^n x}{dt^n} = \int \frac{d^{n+1} x}{dt^{n+1}} \frac{dt}{dT} \, dT$$

or

$$\int \frac{d^{n+1}x}{dt^{n+1}} \, d\tau = \frac{1}{K} \frac{d^n x}{dt^n} \, .$$

Hence, upon integration the scale factor of every variable is multiplied by $1/K$ in addition to the normal gain of the integrator $\left(\frac{1}{RC}\right)$. Changing the normal gain of all integrators by a factor F changes the solution time by a factor $1/F$.

As a simple example, take the system

$$\dot{x} = -ax = -10^{-6}x$$
$$x_0 = 1000$$
$$0 < t < 2 \times 10^6$$

using a schematic similar to that of Figure 23. For a solution time of 25 seconds

$$\frac{dt}{d\tau} = K = \frac{2 \times 10^6}{25} = 8 \times 10^4.$$

Since $x_{max} = 1000$, let $S_x = 0.1$ volt/unit. The gain of the potentiometer and integrator combination must be Ka to traverse the loop and return to the output of the integrator with the originally determined scale factor. The resultant factor of $8 \times 10^4 \times 10^{-6} = 0.08$ can be obtained with a 0.8 potentiometer setting and a 10 microfarad feedback condenser for the integrator. A convenient value for S_t would be 0.5×10^{-4} volts/sec., which would allow t to range from zero to 100 volts during the computation. The overall gain of the integrator generating time with a 100 volt input must be $\frac{0.5 \times 10^{-4} \times 8 \times 10^4}{100} = 0.04$ which can be achieved with a 2.5 megohm input resistor and a 10 microfarad feedback condenser.

As an example of a problem involving several inter-connected loops, Figure 40 is a schematic for the solution of van der Pol's Equation

$$\ddot{x} = \int \dot{x}(1 - x^2) - x.$$

The nature of multiplication is such that greater accuracy results when the larger of the two variables to be multiplied drives the servo and the smaller is across the multiplying potentiometer. If this rule is not followed, large errors can result.

Since the graph paper for the tables is lined 10 x 10 to the half inch, scale factors of inches per unit are convenient for graphical variables. On the large tables one volt of drum rotation corresponds to a $\frac{15}{V}$ inch change on the x-axis of the graph paper with a voltage difference of V volts placed across the drum servo follow-up potentiometer. In the dependent variable axis one volt corresponds to $\frac{10}{V}$ inch on the graph paper with a voltage difference of V volts across the pen servo follow-up potentiometer on the output table. A scale factor of S_i units per inch for the dependent variable on a large input table having x with a scale factor of S_x across the linear slide-wire with one end grounded yields an output with a scale factor of $\frac{S_i S_x}{10}$.

The modifications of the above arguments for the small input tables are straightforward.
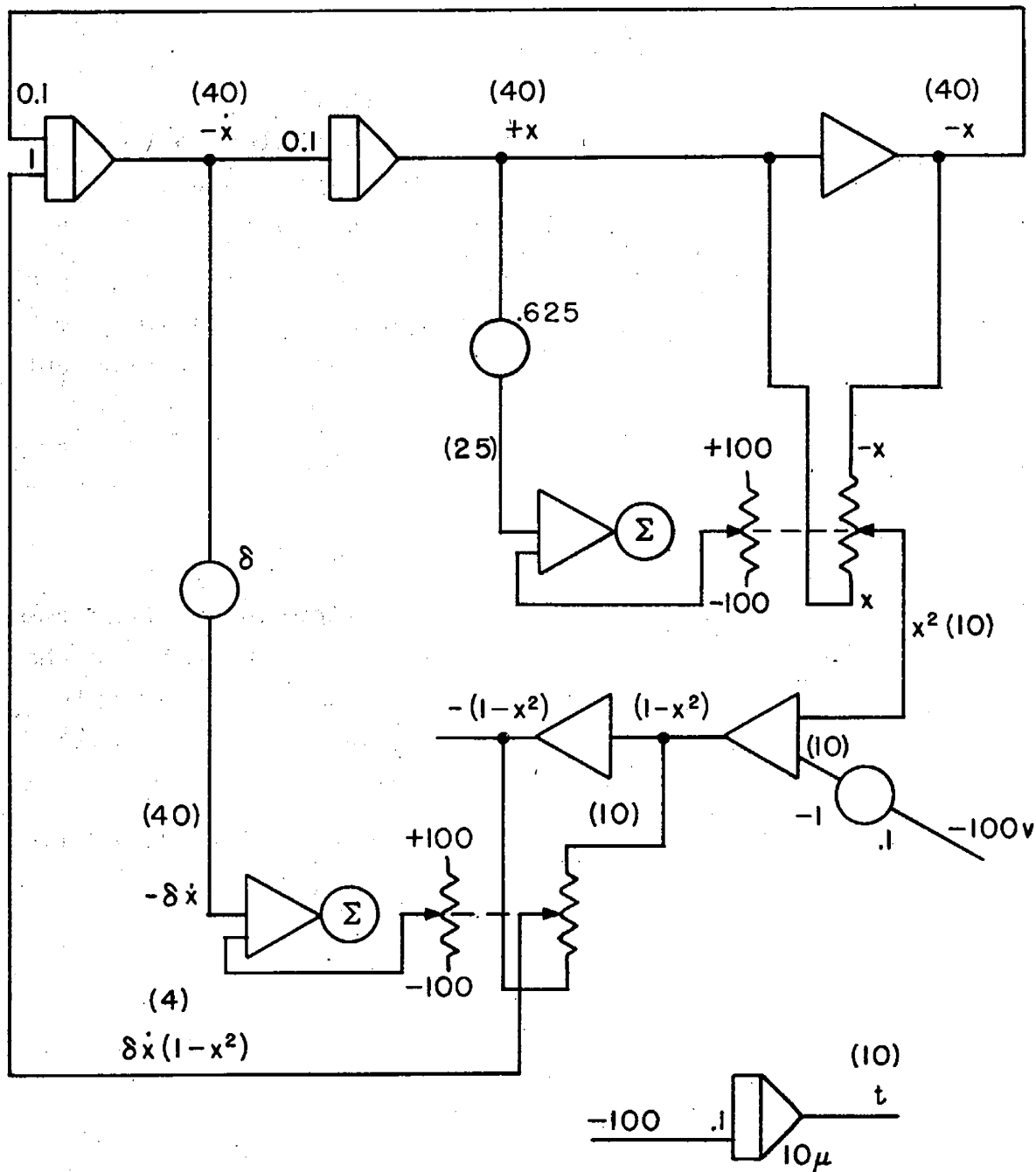
$$\ddot{x} = \delta\dot{x}(1-x^2) - x$$

$$\frac{dt}{d\tau} = K = 0.1$$

Scale factors for δ small are shown in ( ) volt/unit

Fig. 40 — Circuit diagram for Van der Pol equation ($\delta < 1$)

D. Special Servo Applications

One of the greatest advantages of having the servo follow-up potentiometer taps brought to the plugboard is that grounding of one end is possible when the multiplier is always of the same sign. This technique, illustrated in Figure 41, not only eliminates the requirement of a sign-changing amplifier for each multiplying potentiometer, but also reduces the loading on the driving amplifier and improves the accuracy of multiplication.

Placing the divisor voltages across the follow-up potentiometer permits division by the servo when it is desired to have the quotient on the servo shaft, as shown in Figure 42. Actually, this circuit can be considered as a high gain servo-amplifier having $z = x_1/x_2$ as an output and $zx_2 - x_1 = 0$ as an input. The system is not stable unless x_2 is greater than zero, and it has the obvious scale factor restriction that x_2 must be larger than the absolute magnitude of x_1.

All servo potentiometers have center-taps leading to jacks on the plugboard. Grounding these taps improves the accuracy of multiplication when a servo is used in its normal fashion. The taps provide an alternate method of eliminating the need for sign-changing amplifiers when a multiplier is always of one sign. Center-tapped potentiometers also permit easy determination of the absolute magnitude of a variable as illustrated in Figure 43. Figure 44 demonstrates the use of a center-tapped potentiometer as a limiter.

The summing property of the servo amplifiers has proved helpful not only in reducing the need for summing amplifiers, but also for augmenting the number of summing amplifiers for large problems. The output comes from the arm of the follow-up potentiometer and to prevent extreme currents usually no more than a 10 K load (two potentiometers) should be used. A servo so loaded cannot be used for multiplication unless the loading is compensated as described in Section III J.
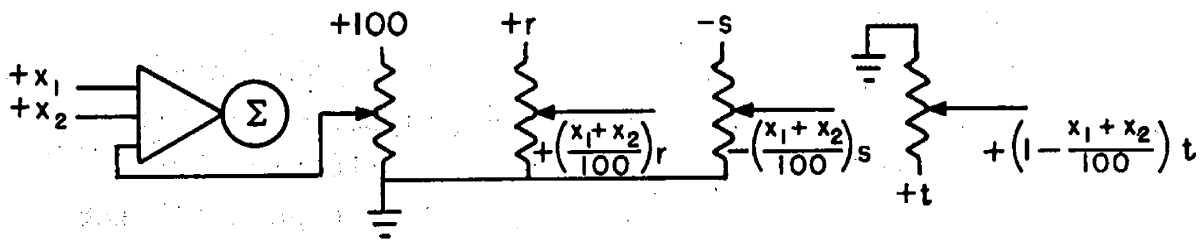
(a) Circuit for $(x_1 + x_2) < 0$
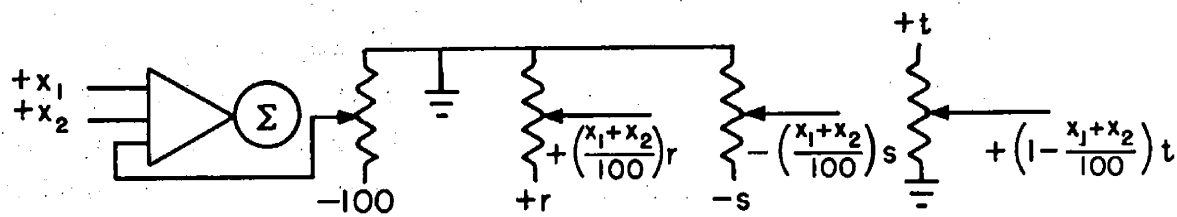
(b) Circuit for $(x_1 + x_2) > 0$

Fig. 41—Servo schematic for multiplier always of same sign
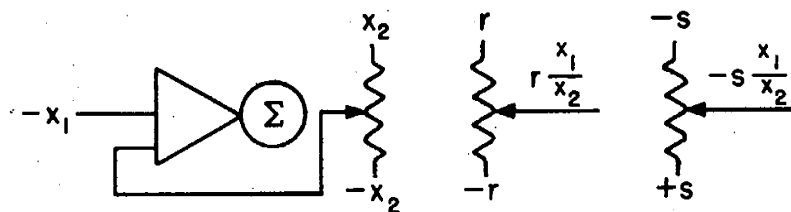
Fig. 42—Simultaneous multiplication and division
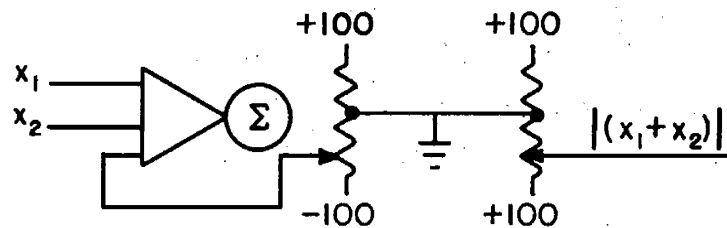
Fig. 43—Determination of the absolute magnitude of a variable
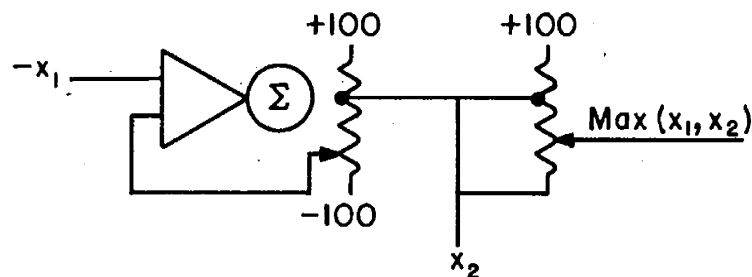
Fig. 44—Limiting with a servo

The accuracy of multiplication can be improved considerably by the proper placement of trimming resistors across the eleven taps provided on each RAND servo potentiometer. While theoretically it is possible to trim each potentiometer to zero error at seven points, it turns out in practice that two to four points usually suffice. A further improvement in accuracy is possible using an input table in conjunction with a servo. In this scheme the error curve between the follow-up and the multiplying potentiometer is plotted and placed on a small servo-rack input table to correct for the error, as shown in Figure 45. A modification of this scheme for correcting voltmeter servo readings is illustrated in Figure 46. This scheme is used in the servo driving the digital readout equipment.
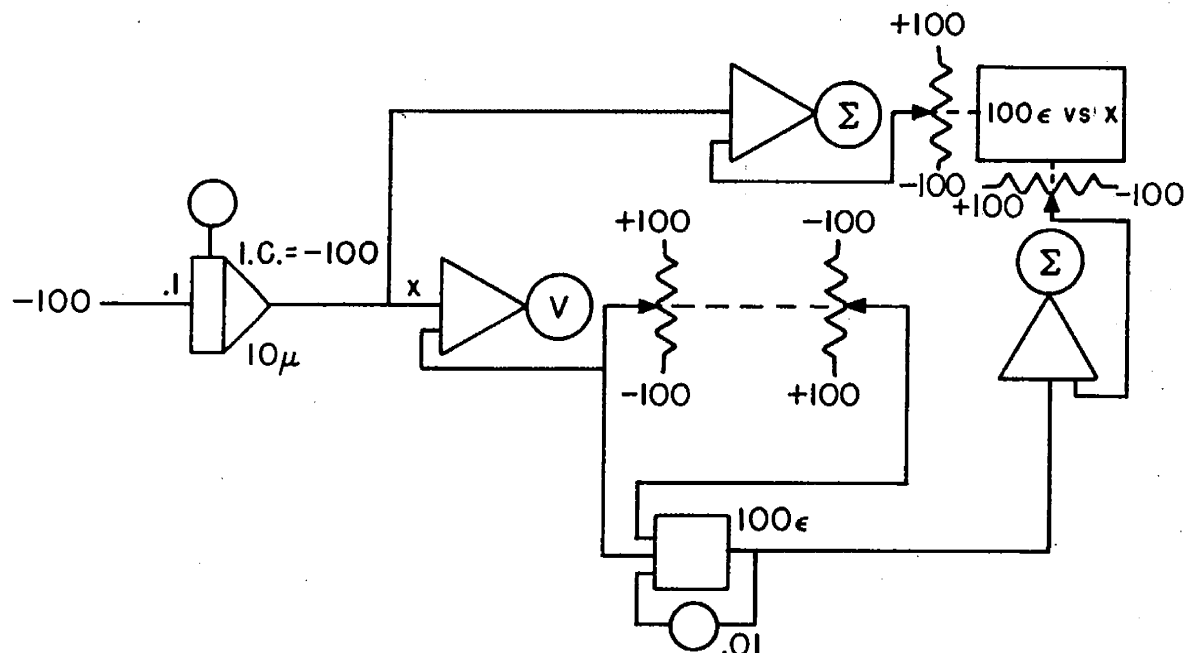
(a) Circuit for plotting relative error
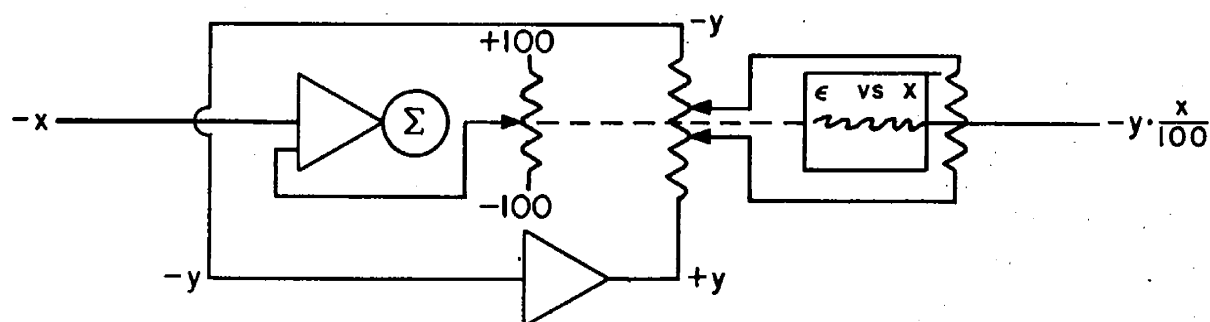
(b) High accuracy multiplication with special two arm potentiometer
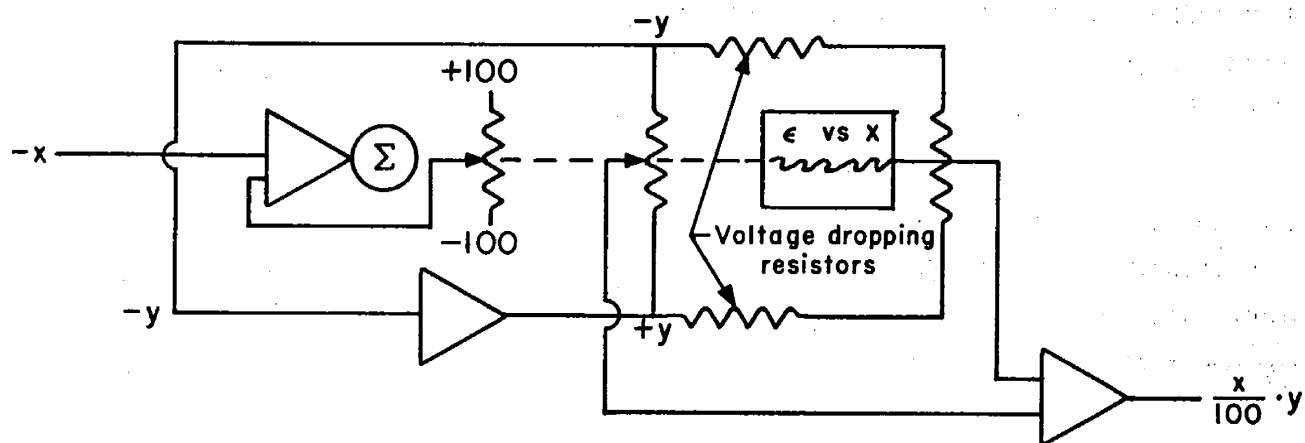
(c) High accuracy multiplication with ordinary potentiometer

Fig. 45—Schematics for high accuracy multiplication

E. Implicit Function Techniques

The theory of implicit function techniques already has been mentioned in Section II A 4 where it was shown that the feedback loop of high gain amplifiers can be closed in such a manner as to solve an implicit function by forcing the sum of the amplifier inputs to zero.

Wherever there is a feedback-loop, there exists the possibility that the feedback signal will be of such a sign as to cause instability. If the circuit is solving $F(z) = 0$, it can be shown that with the output of the high gain amplifier plus z, the circuit will be stable with plus $F(z)$ as the input if $\frac{\partial F}{\partial z} > 0$ in the region of interest. If $\frac{\partial F}{\partial z} < 0$, minus F must be fed back to the high gain amplifier to assure stable operation. These conditions are illustrated in Figure 47. Stable operation results in a similar manner if z or $\int z dt$ replaces z in the output of the amplifier. In this manner integration may be performed by differentiation, and differentiation performed by integration, using the implicit equations

$$z - \int \frac{dz}{dt}\, dt = 0 \quad \text{and} \quad z - \frac{d}{dt}\left(\int z dt \right) = 0.$$

A simple integrating amplifier is an example since the current fed back is proportional to the derivative of the output voltage and the signal current is proportional to the input voltage.

Problems sometimes are encountered in which $\frac{\partial F}{\partial z}$ may be of either sign. Stable operation is possible in this case by minimizing F^2 instead of setting F equal to zero. This is accomplished by solving for the root of the implicit equation $\frac{\partial F}{\partial z} \cdot F = 0.$
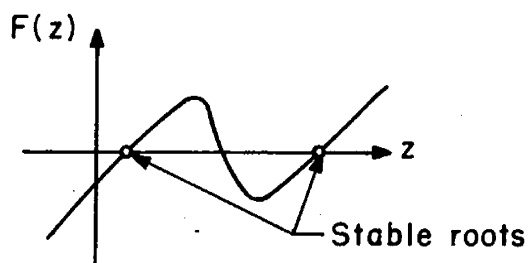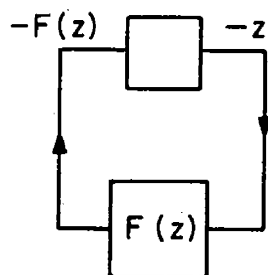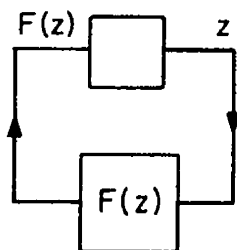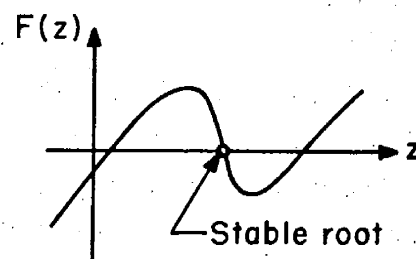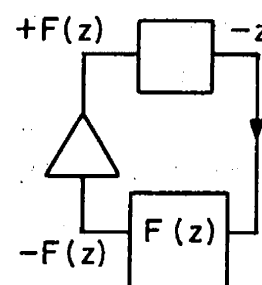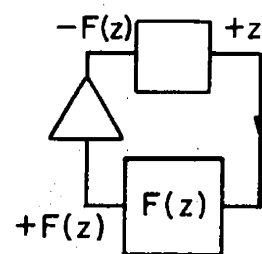
(a) Conditions for $\dfrac{\partial F}{\partial z} > 0$ (b) Conditions for $\dfrac{\partial F}{\partial z} < 0$

Fig. 47   Stability conditions for solution of an implicit function

Where n feedback loops are used to solve a set of simultaneous equations by feeding plus F_i into n high gain amplifiers, the characteristic roots of the Jacobian of the function must be positive for stable operation.

If an analytic analysis is not feasible, less elegant methods may be employed to determine the stability of a system. An odd number of amplifiers in the feedback loop assures the stability of that individual system. Another check is to trace an assumed $+\varepsilon$ error in z through the feedback loop. If the result is a decrease in ε, the system is stable, but a resultant increase in ε indicates instability.

It has been assumed in the above remarks that μ is a constant. Since in practice the gain of an amplifier must always decrease at high frequencies due to inevitable capacities, a rigorous determination of stability must take the variation of μ into account. Unwanted changes in μ produce a phase shift of 45° at about 0.006 c.p.s. when the chopper amplifier is in the loop. The zero frequency gain for this condition is about $|\mu| \approx 6 \times 10^7$. When the basic amplifier only is present $\left(|\mu| \approx 3 \times 10^4 \text{ at zero frequency}\right)$, the phase shift becomes 45° at about 10 c.p.s. It is sometimes found that the parasitic phase shift of the high gain amplifier will cause a system to oscillate when analysis assuming that μ is constant predicts stability. In such cases, often it is possible to "sneak up" on a solution by starting with a small value of effective gain and gradually increasing the gain to the verge of system instability. The error can be determined by measuring the input to the high-gain amplifier. If oscillations still persist or errors are large, the high gain amplifiers of Figure 48 should be tried with the gain as high as possible (k a minimum) with stability. These circuits have a finite delay time, but if the solution time is lengthened sufficiently to keep the high frequency components small, accurate answers should result.
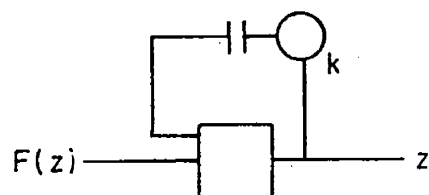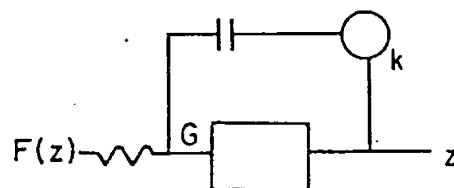
(a) High gain amplifier   (b) Operational high gain integrator

Fig. 48—High gain amplifier modified to improve stability
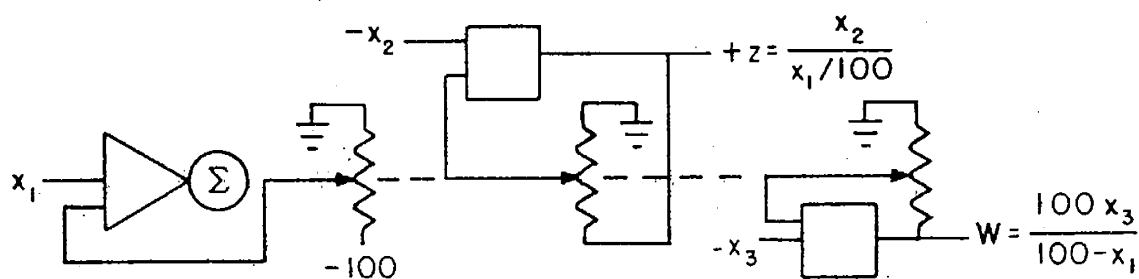
Fig. 49—Circuit for division
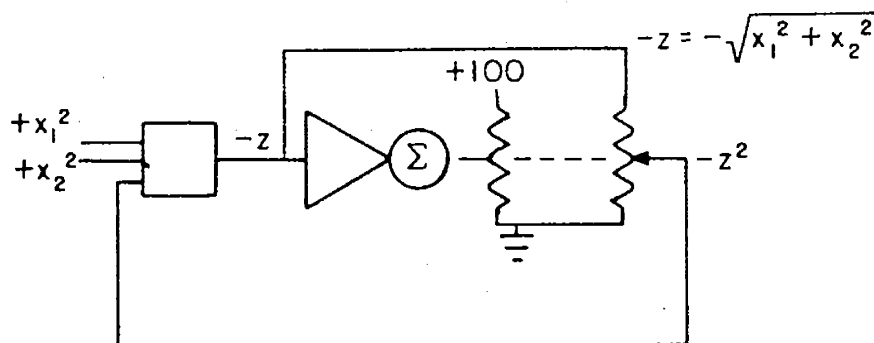
Fig. 50—Circuit for computing a square-root

Division may be accomplished by multiplication through the use of implicit function techniques. The explicit function $z = Z(x_1, x_2) = \frac{x_1}{x_2}$ is put into the implicit form $F(x_1, x_2, z) = x_2 z - x_1 = 0$. Figure 49 shows the schematic diagram for the REAC solution of this equation. In this case $\frac{\partial F}{\partial z} = \frac{\partial (x_2 z - x_1)}{\partial z} = x_2$ and the system is stable for positive x_2 only. Since x_2 will never be negative, the upper tap of each potentiometer is grounded, eliminating the need for a sign-changing amplifier and improving the accuracy.

Implicit function techniques permit the computation of a square-root by squaring. For example, instead of solving $z = \sqrt{x_1^2 + x_2^2}$, we make use of the equivalent relationship $F = z^2 - x_1^2 - x_2^2 = 0$. The schematic diagram for the solution of this equation is shown in Figure 50. The system will be stable for z greater than zero since $\frac{\partial F}{\partial z} = \frac{\partial}{\partial z} \left(z^2 - x_1^2 - x_2^2 \right) = 2z$. Notice that there is only one amplifier in the feedback loop, although at first glance the servo amplifier may appear to be included in the loop.

Implicit function techniques were helpful in the solution of the equations

$$x_1 = e^{-ax_2 x_3}$$

$$x_2 = e^{-\frac{ax_1}{x_3}}$$

where x_1 and x_2 were desired as functions of x_3. The computing loop solved the equation

$$F = \frac{1}{a} x_3 \ln x_2 + e^{-ax_2 x_3} = 0$$

with x_2 the independent variable as shown in Figure 51. A modified high gain amplifier was required to prevent oscillations. Note that

$$\frac{\partial F}{\partial x_3} = \frac{1}{a} \ln x_2 - ax_2 e^{-ax_2 x_3} < 0.$$
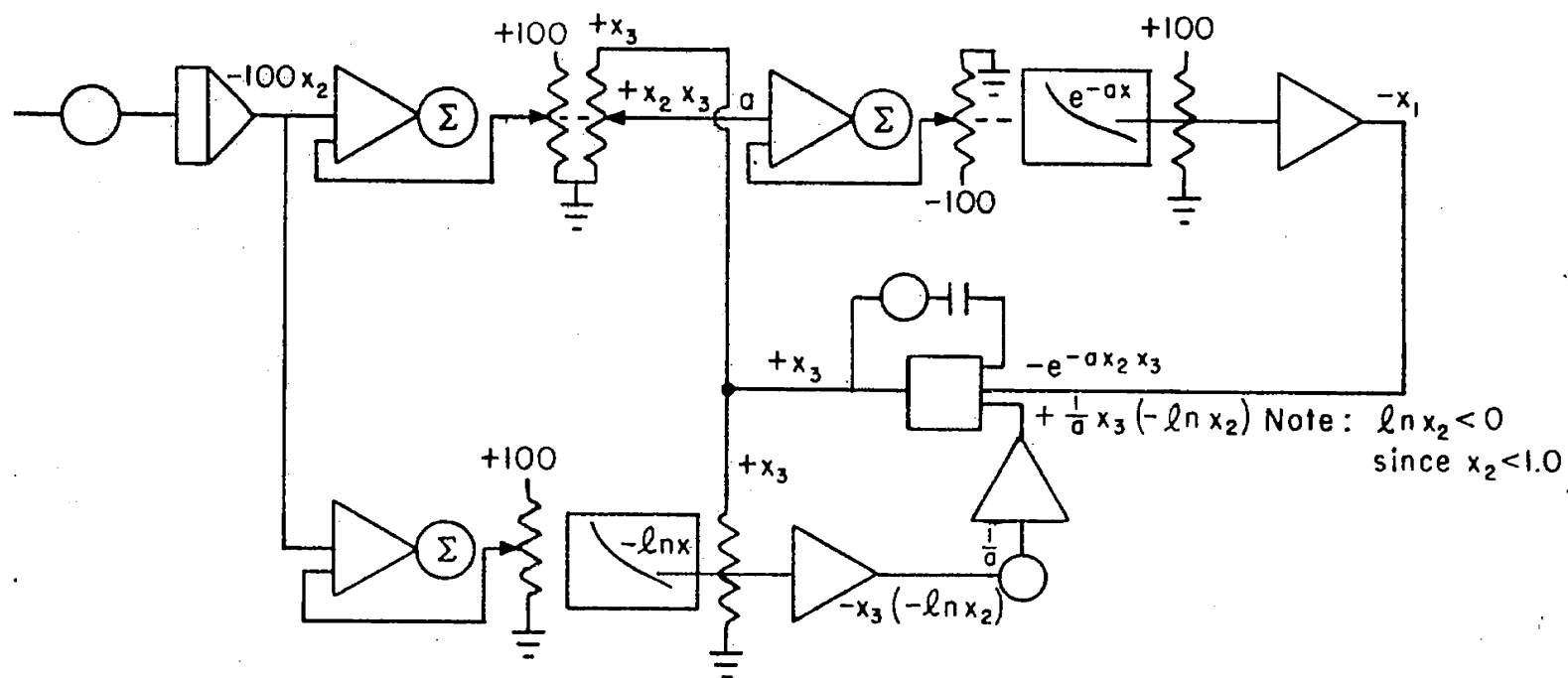
$$F = e^{-ax_2 \, x_3} + \frac{1}{a} x_3 \, \ell n x_2 = 0$$

$$\frac{\partial F}{\partial x_3} = -a x_3 \, e^{-a x_2 \, x_3} + \frac{1}{a} \ell n x_2 < 0$$

Fig. 51—Circuit for solution of $\quad x_1 = e^{-ax_2 \, x_3}$

$$x_2 = e^{-\frac{a x_1}{x_3}}$$
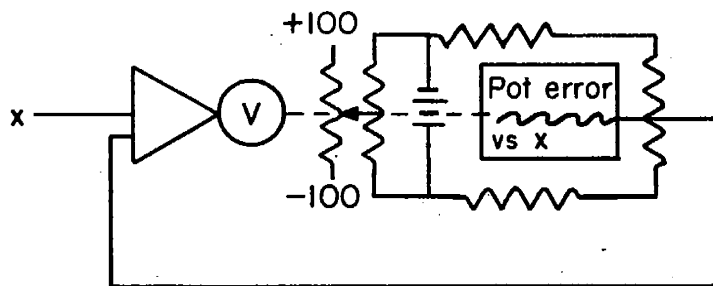
Fig. 46—Schematic diagram for high accuracy servo voltmeter

F. Generation of Functions

Functions are introduced into the REAC either by input tables or by the actual generation of the function by the REAC itself. REAC function generation usually depends upon the solution of a differential equation for which the function is the solution or the approximation of the function by a polynomial or rational function.

Many frequently used analytic functions may be reduced to differential equations readily solved on the REAC. Circuits for several of the more useful equations are given in Figures 52 to 54. These circuits apply only when the variable of integration is the independent variable. Should the variable of integration be another variable, the relationship

$$\int f(x)dx = \int f(x) \frac{dx}{dt} dt$$

must be used, and every integration preceded with a multiplication by $\frac{dx}{dt}$. If $\frac{dx}{dt}$ is not available, it can be generated by the circuit of Figure 55. Ideally the setting of the feedback potentiometer should be zero, but if any of the components of x come off an input table slide wire or a servo potentiometer, the gain of the high gain amplifier must be reduced to prevent high frequency components from saturating the circuit. A proposed schematic diagram for a generalized integrator is presented in Figure 56.

Figure 57 illustrates the generation of sin θ and cos θ using $\frac{d\theta}{dt}$ derived from a high gain circuit forcing y cos θ - x sin θ towards zero to solve the explicit equation $\theta = \tan^{-1} y/x$. The high gain servo is particularly adapted to this application.

Series approximations of a function will usually depend upon one of the following techniques:
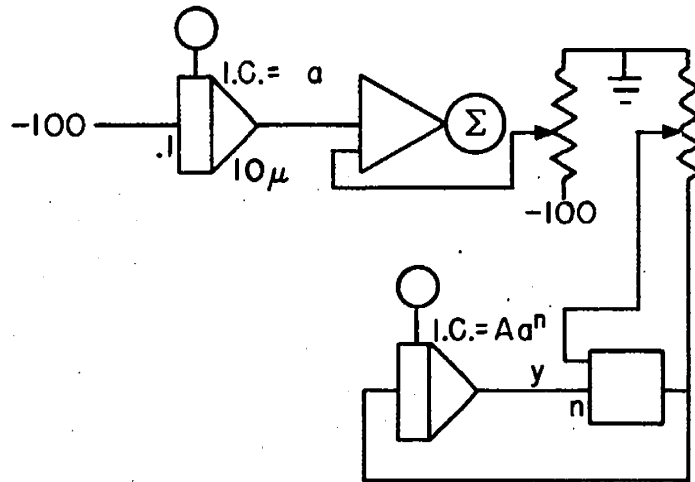
(a) $y = A(t+a)^n$

$(t+a)\,\dot{y} = n\,y$

$y_0 = A a^n$

(b) $y = \dfrac{1}{t+a}$

$\dot{y} = -y^2$

$y_0 = \dfrac{1}{a}$

(c) $z = \ell n(t+a)$
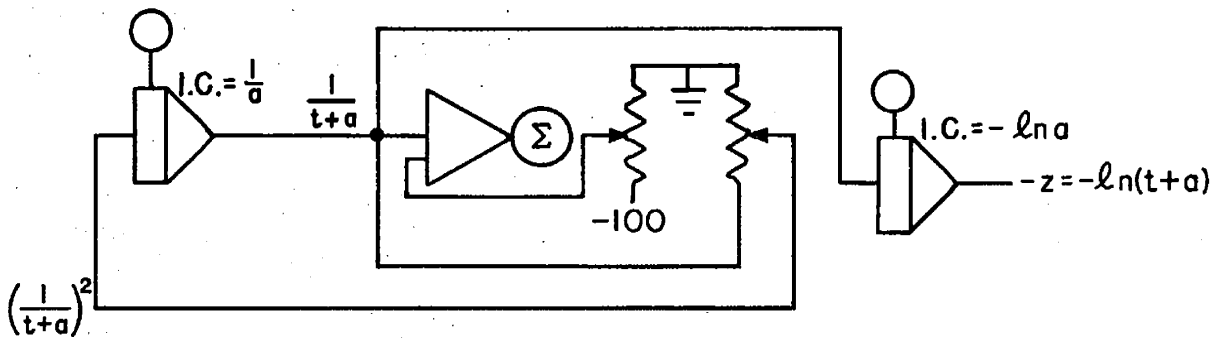
$\dot{z} = \dfrac{1}{t+a}$

$z_0 = \ell n\, a$

Fig. 52 — Circuits for generating $(t+a)^n$ and $\ell n(t+a)$
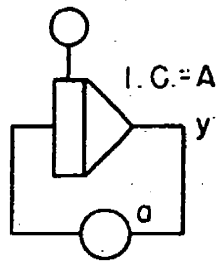
(a)　$y = Ae^{-at}$

$\dot{y} = -ay$

$y_0 = A$
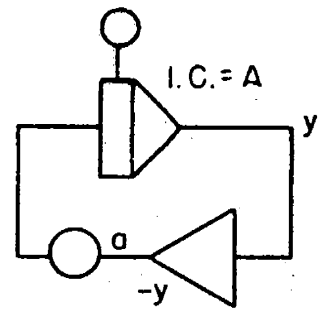
(b)　$y = Ae^{+at}$

$\dot{y} = +ay$

$y_0 = A$
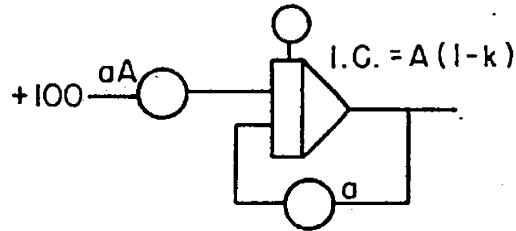
(c)　$y = A(1-ke^{-at})$

$\dot{y} = a(A-y)$

$y_0 = A(1-k)$
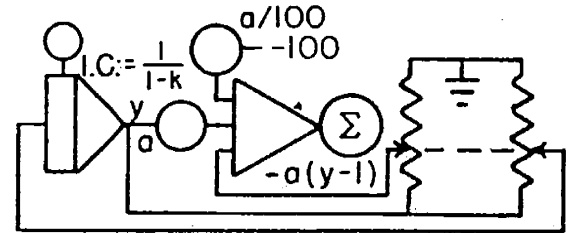
(d)　$y = \dfrac{1}{1-ke^{-at}}$

$\dot{y} = -ay(y-1)$

$y_0 = \dfrac{1}{1-k}$
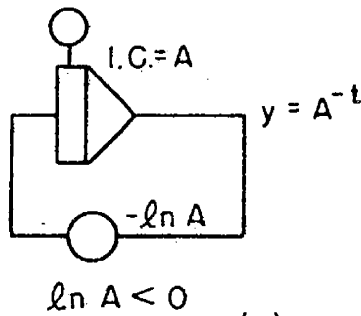
$\ln A < 0$

(e)　$y = A^t$

$\dot{y} = \ln Ay$

$y_0 = A$
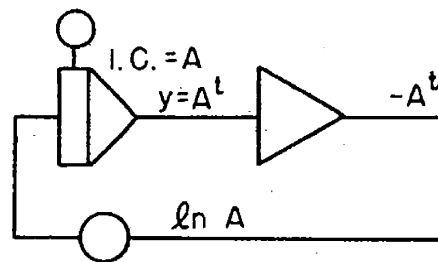
$\ln A > 0$
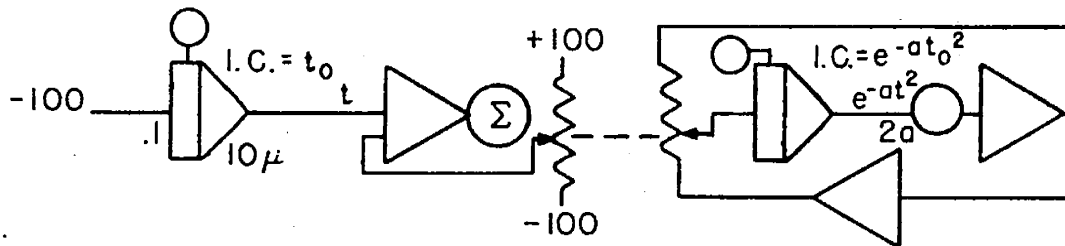
(f)　$y = e^{-at^2}$

$\dot{y} = -2aty$

$y_0 = e^{-at_0^2}$

Fig. 53 — Basic exponential circuits
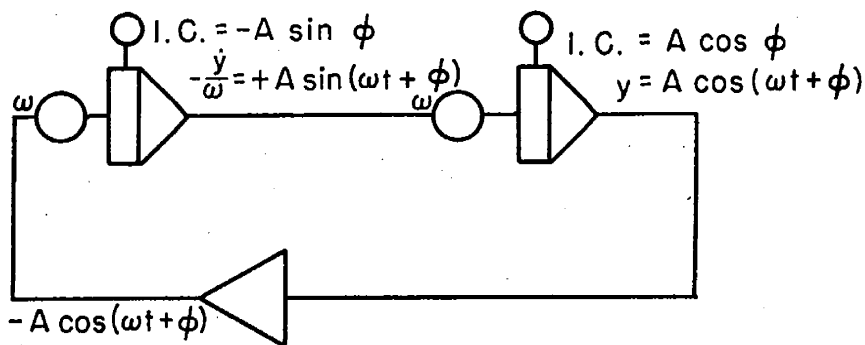
(a) $y = A \cos (\omega t + \phi)$

$\dot{y} = -A\omega \sin (\omega t + \phi)$

$\ddot{y} = -\omega^2 y$

$y_0 = A \cos \phi$

$\dfrac{\dot{y}_0}{\omega} = -A \sin \phi$
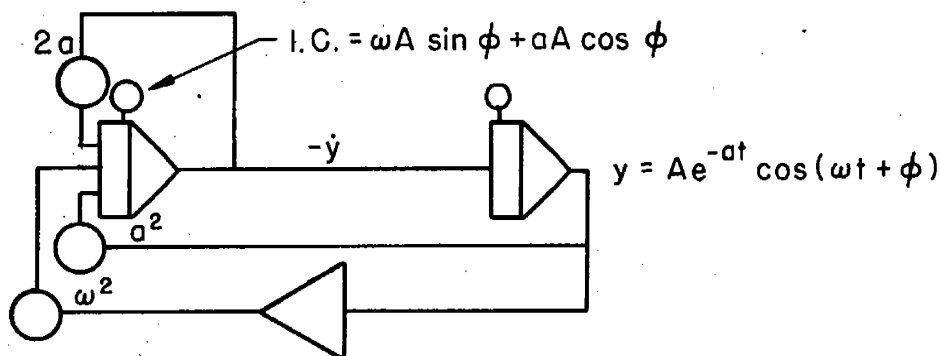
(b) $y = Ae^{-at} \cos (\omega t + \phi)$

$\ddot{y} + 2a\dot{y} + (\omega^2 - a^2) y = 0$

$y_0 = A \cos \phi$

$\dot{y}_0 = -\omega A \sin \phi - aA \cos \phi$

(c) $y = Ae^{-at} \sin (\omega t + \phi)$

$\ddot{y} + 2a\dot{y} + (\omega^2 - a^2) y = 0$

$y_0 = A \sin \phi$

$\dot{y}_0 = +\omega A \cos \phi - aA \sin \phi$

Fig. 54 —— Basic trigonometric circuits
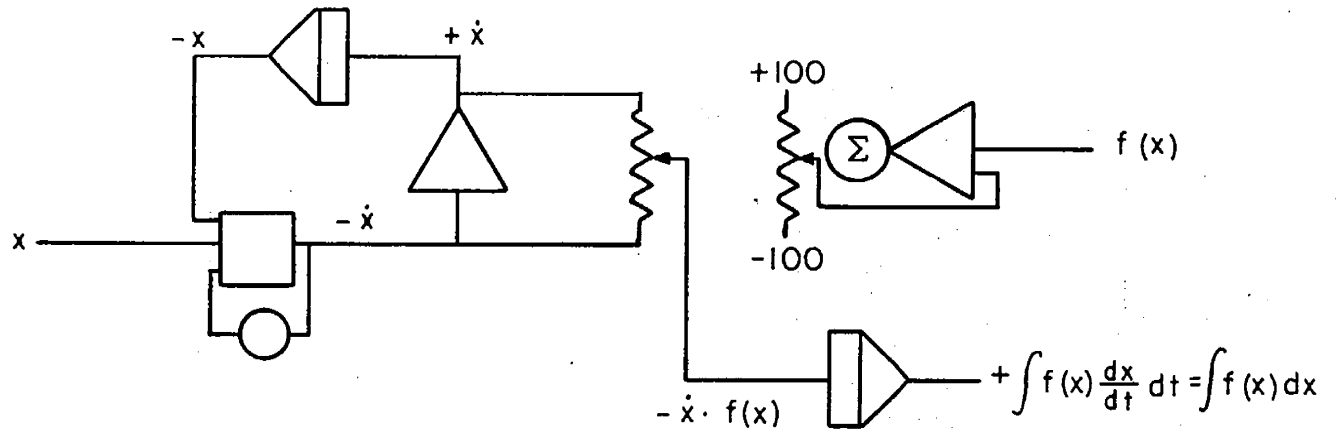
Fig. 55 —— Circuit for generalized integration
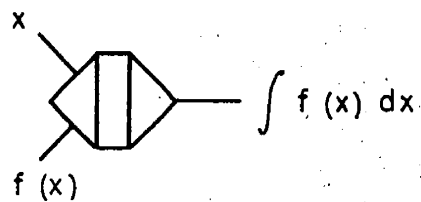
Fig. 56 —— Schematic diagram for generalized integrator
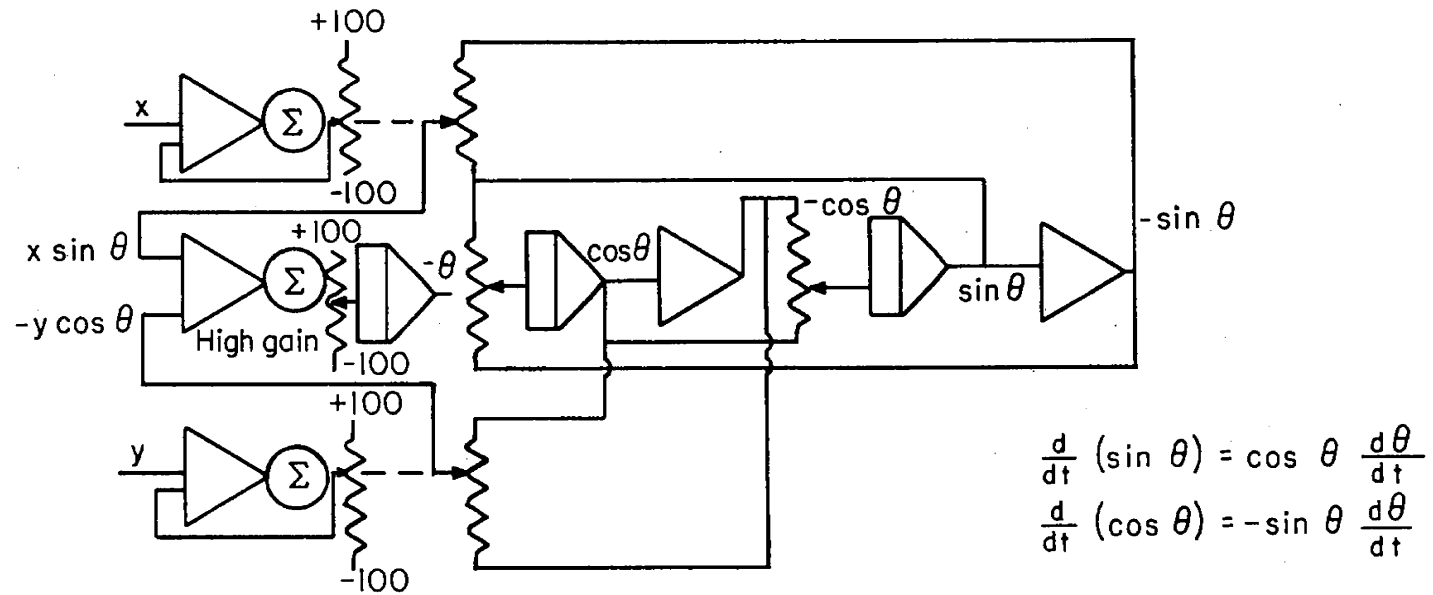
Fig. 57 — Circuit for computing $\theta = \tan^{-1} \frac{y}{x}$, $\sin \theta$, and $\cos \theta$

Within the figure:

$+100$

x

Σ

-100

x sin θ

$+100$

Σ

$-y \cos \theta$

High gain

-100

$-\theta$

$\cos\theta$

$-\cos \theta$

$\sin \theta$

$-\sin \theta$

y

$+100$

Σ

-100

$\frac{d}{dt} (\sin \theta) = \cos \theta \, \frac{d\theta}{dt}$

$\frac{d}{dt} (\cos \theta) = -\sin \theta \, \frac{d\theta}{dt}$

$F = x \sin \theta - y \cos \theta = 0$

$\frac{\partial F}{\partial \theta} = x \cos \theta + y \sin \theta = \sqrt{x^2 + y^2} > 0$

a) Taylor series – for the approximation of analytic function in a small region.

b) Lagranges' formula – for the computation of an nth degree curve passing through n+1 arbitrary points.

c) Finite differences equations – for the computation of an nth degree curve passing through n+1 points with equal spacing.

d) Rational fraction or reciprocal difference equations – for the representation of a function in a region near a pole.

e) Least squares approximation – for minimizing the error in passing an nth degree polynomial through m points with n < (m+1).

Milne's "Numerical Calculus" and Hastings' "Treatise on Rational Function Approximation" give good treatments of the above techniques.

Several useful series and their ranges for various inaccuracies are listed in Table V.

Table VI lists the Tchevycheff approximations to x^n by a polynomial of power n-1 for 0 < x < +1 and -1 < x < +1. In these formulae, $T_i(x)$ and $\overline{T}_i(x)$ always lie between plus and minus unity, which makes the maximum error computation straightforward. These formulae are particularly useful for the "economization", or reduction, of series of higher accuracy and power than required.

TABLE V. SOME USEFUL APPROXIMATIONS

Approximation	Error	Limit
1) $(1 \pm x)^m \doteq 1 \pm mx$	ε	$|x^2| < \dfrac{2\varepsilon}{m(m-1)}$
2) $e^{\pm x} \doteq 1 \pm x + \dfrac{x^2}{2}$	.01 .001	$|x| < .39$ $|x| < .18$
3) $a^{\pm x} \doteq 1 \pm (x \ln a) + \dfrac{(x \ln a)^2}{2}$	ε	$|x^3| < \dfrac{6\varepsilon}{(\ln a)^3}$
4) $\sin x \doteq x$	.01 .001	$|x| < .31 = 18^0$ $|x| < .15 = 8.5^0$
5) $\sin x \doteq x - \dfrac{x^3}{6}$	.01 .001	$|x| < 1 = 57.3^0$ $|x| < .65 = 37.3^0$
6) $\cos x \doteq 1$	.01 .001	$|x| < .1 = 6^0$ $|x| < .03 = 2^0$
7) $\cos x \doteq 1 - \dfrac{x^2}{2}$	.01 .001	$|x| < .70 = 40^0$ $|x| < .39 = 22.5^0$
8) $\tan x \doteq x$	.01 .001	$|x| < .24 = 14^0$ $|x| < .11 = 6.3^0$
9) $\tan x \doteq x + \dfrac{x^3}{3}$	.01 .001	$|x| < .59 = 34^0$ $|x| < .37 = 21.3^0$
10) $\sin^{-1} x \doteq x$	.01 .001	$|x| < .31 = 18^0$ $|x| < .15 = 8.5^0$
11) $\sin^{-1} x \doteq x + \dfrac{x^3}{6}$	.01 .001	$|x| < .63 = 36^0$ $|x| < .41 = 23.5^0$
12) $\tan^{-1} x \doteq x$	.01 .001	$|x| < .24 = 14^0$ $|x| < .11 = 6.3^0$
13) $\tan^{-1} x \doteq x - \dfrac{x^3}{3}$	.01 .001	$|x| < .68 = 39^0$ $|x| < .42 = 24^0$

TABLE VI. TCHEVYCHEFF POLYNOMIAL APPROXIMATION

For $0 < x < 1$, $-1 < T_i(x) < +1$

$$x = \frac{1}{2} + \frac{1}{2}T_1(x)$$

$$x^2 = -\frac{1}{8} + x + \frac{T_2(x)}{8}$$

$$x^3 = \frac{1}{32} - \frac{9x}{16} + \frac{3x^2}{2} + \frac{T_3(x)}{32}$$

$$x^4 = -\frac{1}{128} + \frac{32x}{128} - \frac{160x^2}{128} + 2x^3 + \frac{T_4(x)}{128}$$

$$x^5 = \frac{1}{512} - \frac{50x}{512} + \frac{400x^2}{512} - \frac{1120x^3}{512} + \frac{1280x^4}{512} + \frac{T_5(x)}{512}$$

$$x^6 = -\frac{1}{2048} + \frac{72x}{2048} - \frac{840x^2}{2048} + \frac{3584x^3}{2048} - \frac{6912x^4}{2048} + \frac{6144x^5}{2048} + \frac{T_6(x)}{2048}$$

$$x^7 = \frac{1}{8192} - \frac{98x}{8192} + \frac{1568x^2}{8192} - \frac{9480x^3}{8192} + \frac{26880x^4}{8192} - \frac{39424x^5}{8192} + \frac{28672x^6}{8192} + \frac{T_7(x)}{8192}.$$

$-1 < x < +1$, $-1 < \overline{T}_i(x) < +1$

$$x = 0 + \overline{T}_1(x)$$

$$x^2 = \frac{1}{2} + \frac{1}{2}\overline{T}_2(x)$$

$$x^3 = \frac{3x}{4} + \frac{1}{4}\overline{T}_3(x)$$

$$x^4 = x^2 - \frac{1}{8} + \frac{1}{8}\overline{T}_4(x)$$

$$x^5 = \frac{5}{4}x^3 - \frac{5}{16}x + \frac{1}{16}\overline{T}_5(x)$$

$$x^6 = \frac{3}{2}x^4 - \frac{9}{16}x^2 + \frac{1}{32} + \frac{1}{32}\overline{T}_6(x)$$

$$x^7 = \frac{112}{64}x^5 - \frac{56}{64}x^3 + \frac{7}{64}x + \frac{1}{64}\overline{T}_7(x)$$

G. Input Table Applications

In case it is impossible or difficult to generate a function internally on the REAC it is necessary to introduce it into the machine through an automatic input table. Hand tracking may be necessary for certain multiple-valued functions, although use of two tables and relay control may work for some and tracking the x and y coordinates versus arc length for others. As an example of the latter technique, the REAC was "taught" to write its name automatically by plotting the x and y coordinates as a function of arc length so modified as to speed up the writing where the slope was nearly constant and to slow down the writing when the slope changed rapidly.

The installation of automatic input tables has greatly modified the planning of schematics for problems. Previously, input functions were replaced wherever possible by approximations or generation of the functions by the REAC itself. However, since a wired curve for the input table takes little more time to prepare than that required for drawing a curve through the points with French curves, the use of input tables even for simple functions is becoming a standard practice at RAND. The resulting simplification of circuitry reduces scale factor problems and the probability of machine errors, and simplifies any debugging that may be necessary. The following problem illustrates such a use of input tables.

$$\ddot{\rho} - (R + \rho)(\dot{\varphi}_0 + \dot{\varepsilon})^2 + \frac{\mu}{(R + \rho)^2} = B_r \cos \varepsilon$$

$$(R + \rho)\ddot{\varepsilon} + 2(\dot{\varphi}_0 + \dot{\varepsilon})\dot{\rho} = - B_r \sin \varepsilon.$$

Letting $x = \rho/R$, $y = \frac{\varepsilon}{\varphi_0}$ and modifying to a more satisfactory form for computation we obtain:

$$\ddot{x} = \dot{\phi}_0^2 \left\{ x + (1 + x)(2 + \dot{y})\dot{y} + \frac{\mu}{R^3 \dot{\phi}_0^2} \cdot \frac{(2 + x)x}{(1 + x)^2} - \frac{B_r}{R\dot{\phi}_0^2} (1 - \cos \varepsilon) \right\}$$

$$(1 + x)\ddot{y} = - 2(1 + \dot{y})\dot{x} - \frac{B_r}{\phi_0 R} \sin \varepsilon$$

The approximations $\cos \varepsilon = 1 - \frac{\varepsilon^2}{2} + \frac{\varepsilon^4}{24}$ and $\sin \varepsilon = \varepsilon - \frac{\varepsilon^3}{6}$ further simplify the computation and improve the accuracy since modulators and demodulators need not be used. Moreover, the form $(1 - \cos \varepsilon)$, which is poor computationally for small ε, is eliminated.

Since the above system is unstable, caution was necessary in planning the computing procedure and in operating the REAC to meet the request for plots of asymptotic behavior for small initial values of $\ddot{\rho}$, $\dot{\rho}$, ρ, $\ddot{\varepsilon}$, $\dot{\varepsilon}$, and ε.

At first, a solution was attempted computing $\frac{x(2 + x)}{(1 + x)^2}$ and $\frac{1}{1 + x}$, but the drift voltages without driftless d.c. amplifiers made it impossible to use as small starting values as desired and still obtain good accuracy. A second set of runs was made generating $\frac{1}{1 + x}$ and $\frac{x(2 + x)}{(x + 1)^2}$ on input tables. The resultant reduction in the circuitry permitted use of the small initial values desired.

The use of input tables greatly simplifies scale-factor considerations when functions of the form $g(x) \cdot f(x)$ appear, in which the product has reasonable limits but $g(x)$ and $f(x)$ vary considerably. If $f(x)$ and $g(x)$ are computed on the REAC and then the product formed, it will be extremely difficult to select scale factors yielding an accurate answer over the range of x. Since no scale factor troubles will result if an input table is used, such a plan is preferable unless the computation is too tedious.

Since it is possible to have more than one wired curve on an input table if the curves do not intersect, several functions of the same independent variable may be generated by one input table. Curves can be prevented from intersecting by modifying one

or more of the functions, such as adding a constant, changing the
scale-factor, or specifying a portion of the vertical scale of the
graph paper for each function.

Test runs with the automatic generation of a function of
two variables have given encouraging results. Figure 58 illustrates
one circuit that has been used. The second variable was represented
by a three curve parametric plot with the servo linearly interpolating
between the two curves bounding the second variable. The test runs
were restricted to three-family curves since only potentiometers with
three taps were available on the servos at the time. A five family
set of curves has been successfully tracked. Potentiometers with
several taps are to be installed on all servos which will accommodate
functions of two variables requiring more complete definition in the
second variable. The taps on the potentiometer need not be
separated by uniform increments of the winding but the values of
y chosen for the wired curves must, of course, correspond to the
electrical location of the particular taps used. In such a situation,
a switch geared to the servo shaft may be used to allow only three
amplifiers to drive the appropriate points on the tapped servo
potentiometer. For example, referring to Figure 59, suppose
$y_2 \leq y \leq y_3$ and $y \rightarrow y_3$. It is necessary to arrange the switch
controlling amplifier A to transfer from position 1 to position 4
for some y in (y_2, y_3) the exact point (or interval) or transition
being unimportant since the value of $\phi(x,y)$ is determined by the
outputs of the two amplifiers connected to the taps bridging the
particular value of y. Theoretically, it is possible to use only
two amplifiers and a "suitable" switch but the extreme precision
required of switch operation timing would probably make this an
impractical bit of elegance. Alternately, potentiometer loading
compensation techniques may be used. If the parametric curves
cross, additional tables driven by the same independent variable
must be used.
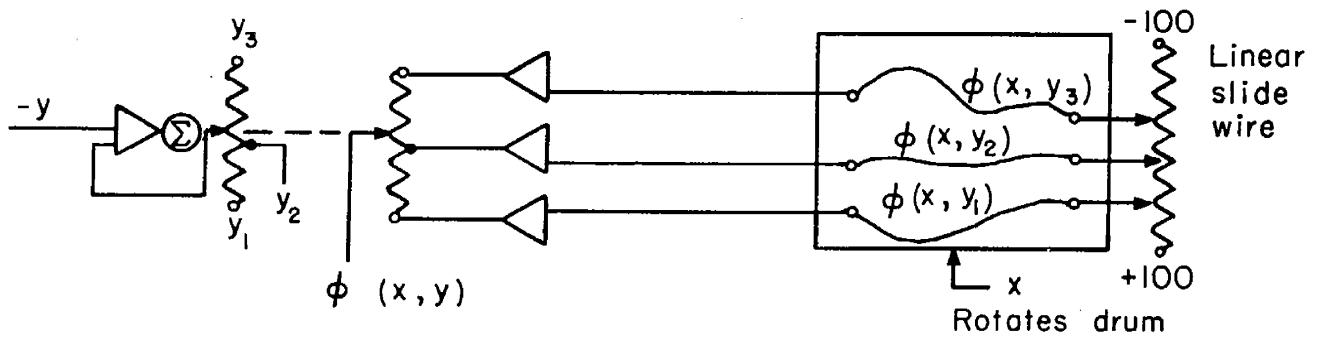
Fig. 58 — Automatic generation of a function of two variables
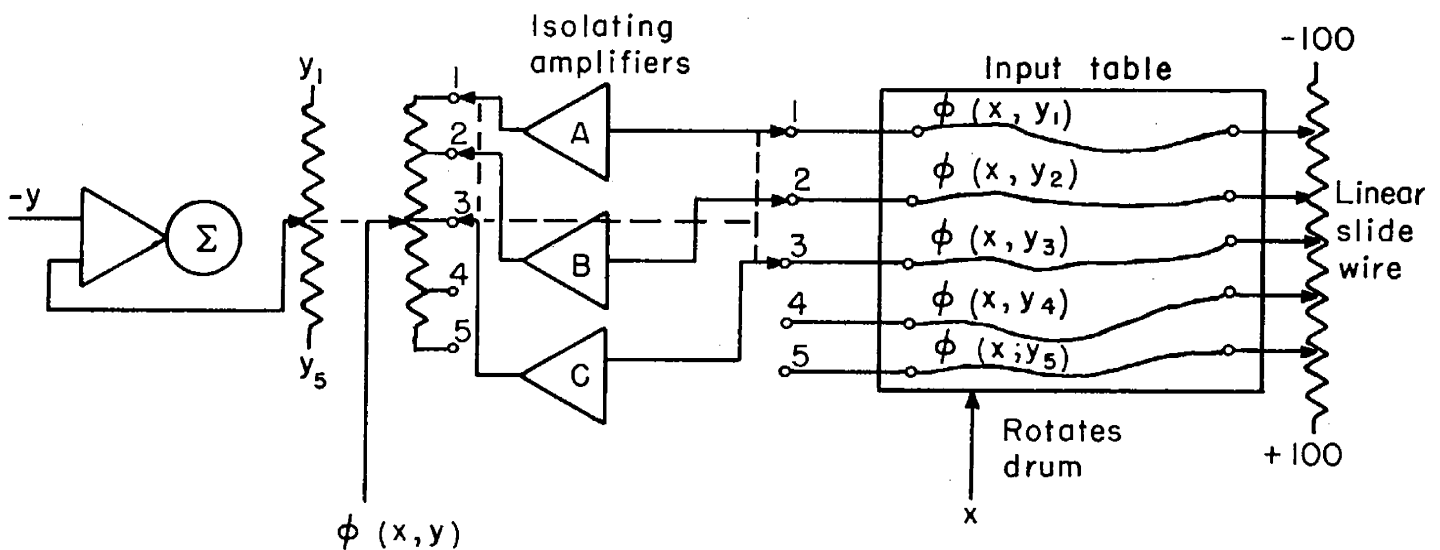
Fig. 59 — Generation of a function of two variables from a five parameter family

The inverse of a function plotted may be found using a high gain servo-amplifier driving the input table. The computation of $\sin^{-1} x$ from a curve of sin x is illustrated in Figure 60.

The small, servo rack input tables will be used when their inaccuracy can be compensated by scale factor reductions in the computations. In particular, they will be useful in adding the correction terms to the first terms of approximations. For example, let sin x = x + $\mathcal{E}(x)$ for small values of x where $\mathcal{E}(x)$ will be a small correction term and will be entered with a large scale-factor which will be reduced before the term is added to x.
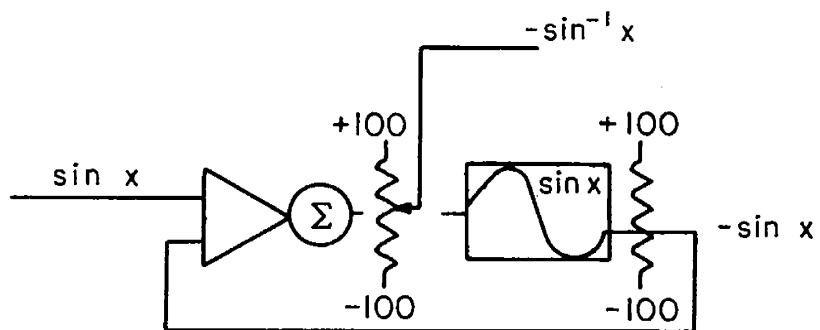
Fig. 60 — Circuit for computing sin⁻¹ x
using a sin x curve

H. Relay Circuits

Relays may be connected from the plug board to open the input to any or all amplifiers. One convenient application is the automatic stopping or modifying of a problem whenever one voltage exceeds another. Figure 61 shows one way in which this may be accomplished. Current will flow through the relay, holding it open until x_1 becomes greater than x_2, at which time the output of the high gain amplifier goes from its maximum positive value of +300 V to its maximum negative value of -190 V, current flow is prevented by the diode of a limiter, and the relay opens. The resistance in series with the relay coil limits the current flow to the proper operating value. The precision of this automatic hold circuit at normal computing speeds has been found to be better than 0.1 per cent over several days of computing. When an amplifier is used in this manner, it must be placed in the manual, rather than automatic, balance position.

The following problem used the control of individual hold relays as a limiting device in the steady-state solution of the equations

$$H_i = \max \left[K \sum_{j=1}^{3} A_{ij} x_i, 0 \right]$$

$$\frac{dx_i}{dt} = H_i - \lambda x_i$$

$$\lambda = \sum_{i=1}^{3} H_i$$

$$A_{ij} = \begin{bmatrix} 0 & \alpha & -\beta \\ -\alpha & 0 & \gamma \\ \beta & -\gamma & 0 \end{bmatrix}$$

for various sets of K, α, β, γ and initial x_i's. The equations for the H_i's were first set up using the limiters provided in the REAC, but the softness of their limiting action caused the variables to oscillate quite badly about their roots. The use of high-gain amplifiers driving the hold relays of amplifiers with H_i's as inputs gave much more satisfactory results. The overloading and high frequency components present at the start of the solution caused the final x_i's to be low although of the proper ratios. Making

$$\lambda = \sum_{i=1}^{3} H_i - \left(1 - \sum_{i=1}^{3} x_i \right)$$

corrected this fault. With K = 4 the oscillations about the steady state solutions dropped to about one per cent in 15 seconds and converged with a 0.2 per cent error after one minute. K's greater than 4 produced instability due to the resulting great overload and high frequency components. Solutions with the $\sum H_i$ term removed from the new λ were as satisfactory as those with the term present.

Many other applications of a hold relay in conjunction with a high-gain amplifier and diode are possible. As an example, consider the computation of the percentage of time one variable x_1 exceeds another variable x_2. A constant voltage is fed to the input of two integrators and the hold relay of the first integrator is opened by an automatic circuit as shown in Figure 62 whenever $x_1 < x_2$. The output of the first integrator divided by the second is the desired ratio. Since the opening and closing times of relays are not identical, the total computing time should be large to keep the resultant error small. A trial run of this scheme gave results that were always within 0.1 per cent of the true value.

As another example, suppose it is required to find the maximum to time t of a function f(t), defined as $f_{max}(t)$. Since the condensers are the only place in the REAC where voltages can be stored, the maximum can appear as the output of an integrator, and f(t) generated as
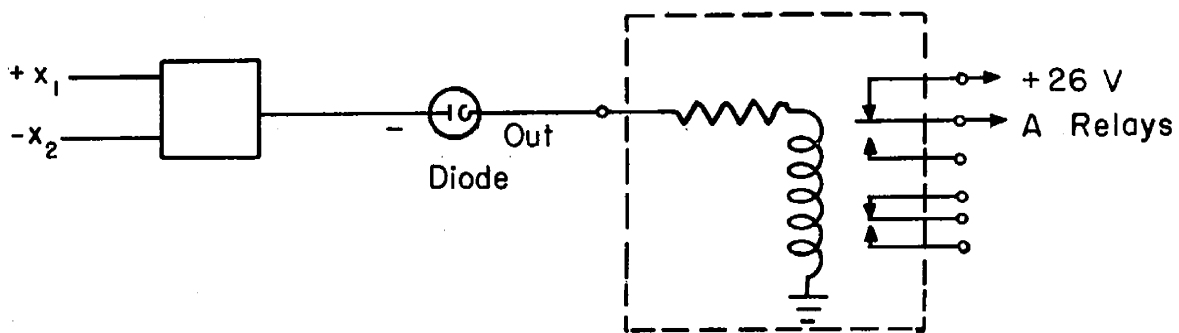
Fig. 61 —— Automatic hold circuit
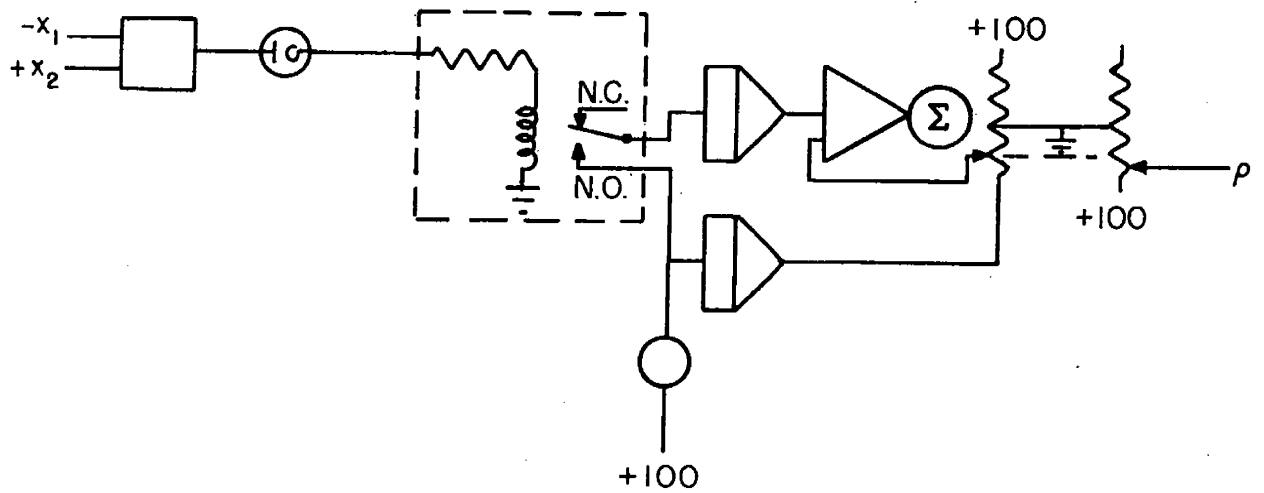
Fig. 62 — Computation of ρ = per cent of time $x_1 > x_2$
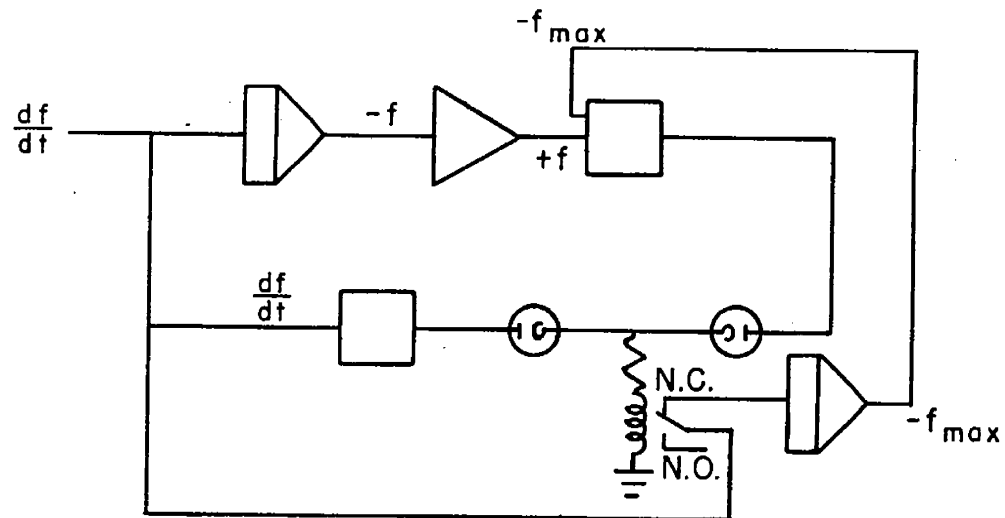
Fig. 63 — Computation of the maximum of a function

$$f(t) = \int_0^t \frac{df}{dt}\, dt.$$

Two high-gain amplifiers and associated equipment can be used to control the hold relay of the integrator generating $f_{max}(t)$ in such a manner that the input df/dt is connected if and only if both $df/dt > 0$ and $f(t) > f_{max}(t)$ as illustrated in Figure 63. The modifications required to obtain a minimum are straightforward.

Figure 64 illustrates how a relay may be used to solve a difference equation of the form

$$\triangle x_i = \Psi_i(x_1, x_2, \ldots, x_n, t); \quad t = 0, k, 2k, 3k, 4k, \ldots$$

The value of the condenser across the servo input is not critical, since it charges through the negligible amplifier dynamic impedance and discharges during the period it is used as a "memory" through the very high impedance of a servo in the voltmeter connection (the current drain in the output is from the follow-up potentiometer of the servo). The relay position will change every $1/K$ seconds. When the relay is not energized, t increases by

$$K \int_0^{K^{-1}} \triangle t\, dt = \triangle t,$$

and the servo moves to the proper $\Psi_i(x_1, x_2, \ldots, x_n, t)$ value. When the relay is energized, x_i increases by

$$K \int_0^{K^{-1}} \triangle x_i\, dt = \triangle x_i,$$

with $\triangle x_i$ and t being held constant during these integrations.

A slightly different relay circuit is shown in Figure 65. The purpose of the circuit was to introduce the data plotted on two
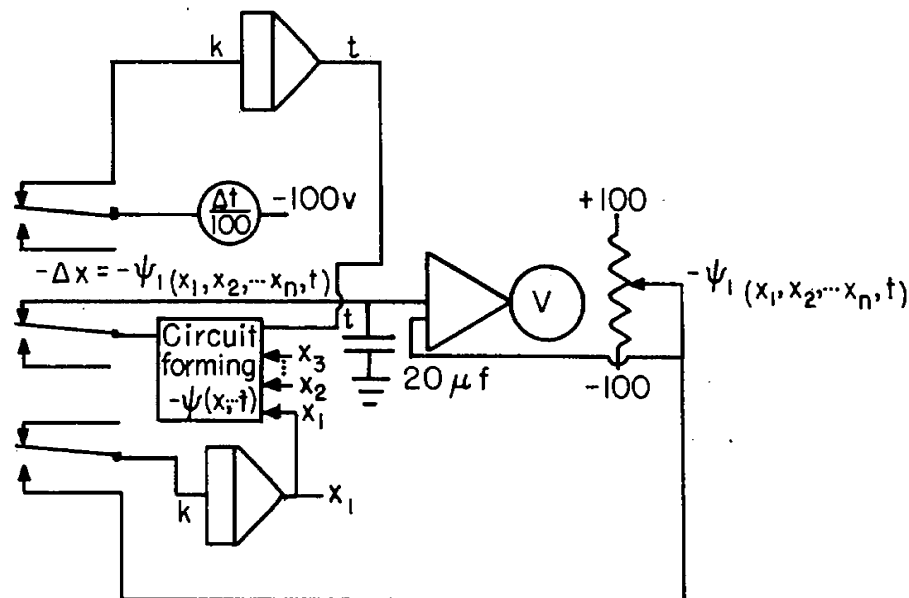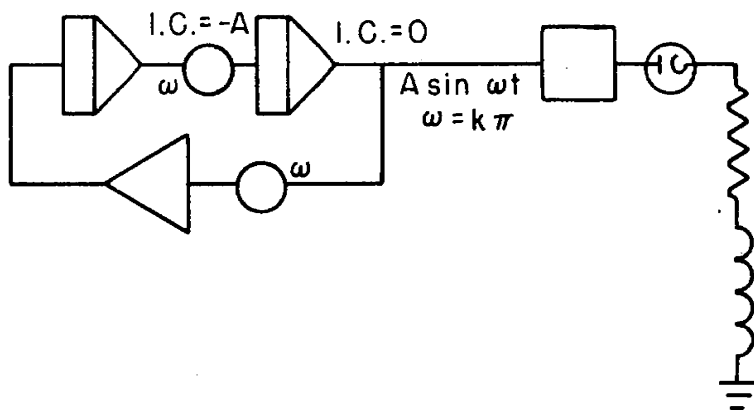
Relay position changes every $\frac{1}{k}$ seconds

When relay is not energized, t increases by $k\int_0^{k^{-1}} (\Delta t)\, dt = \Delta t$, and servo moves to $\psi_1{}_{(x_1, x_2, \cdots x_n, t)}$

When relay is energized, x_1 increases by $k\int_0^{k^{-1}} (\Delta x_1)\, dt = \Delta x_1$, Δx_1 and t being constant during this integration

Fig. 64 — Circuit for solving $\Delta x_1 = \psi_1 (x_1, x_2, \cdots x_n, t)$

input tables in a cyclic manner in both directions. Notice one relay is connected in such a manner that it closes only when the two high gain amplifiers have outputs of opposite signs.

Figures 66 and 67 demonstrate the use of relays in the place of servos in generalized integration and multiplication. The relays are closed $\frac{dx/dt}{100G} \times 100$ per cent of the time and, as a consequence, integration following any relay contact is effectively multiplied by $\frac{dx/dt}{100G}$.
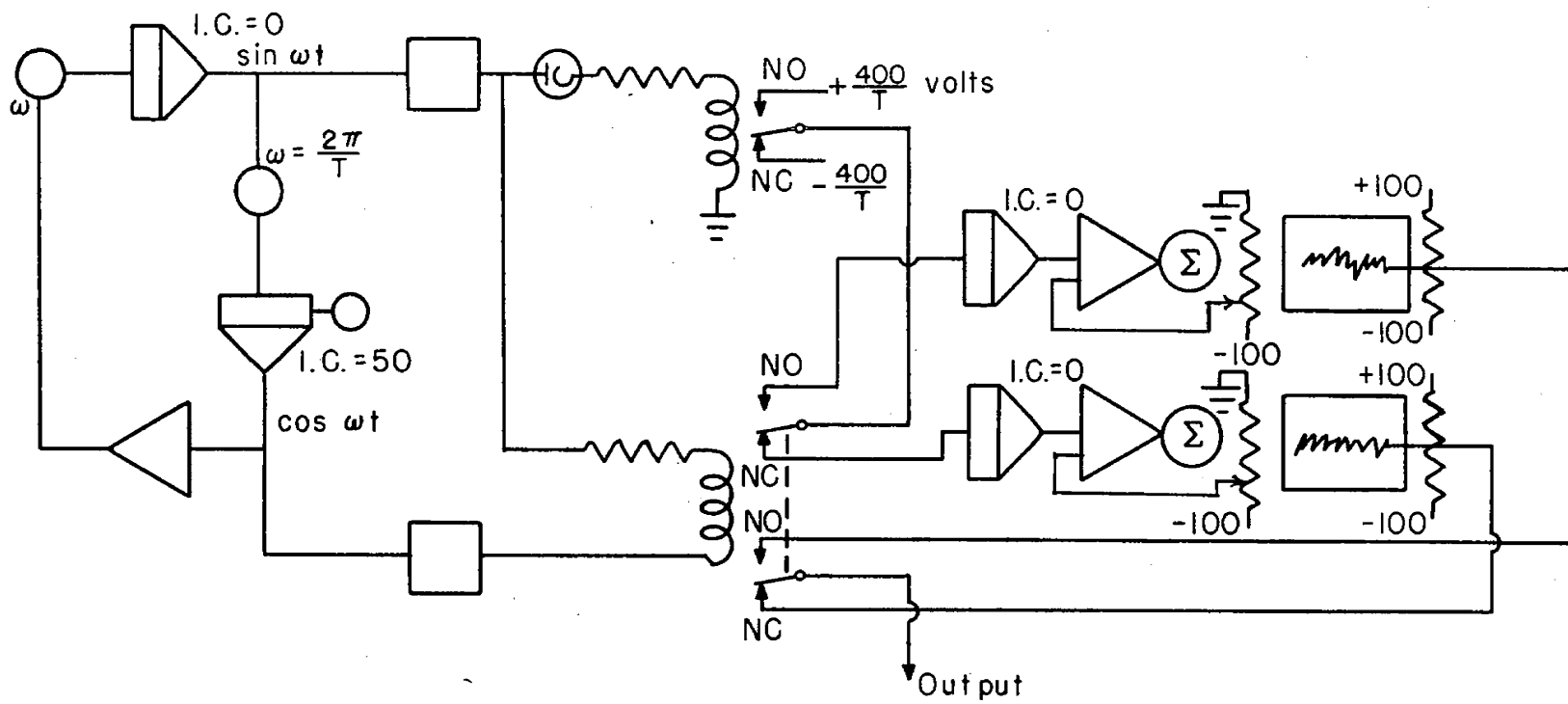
Fig. 65 —Circuit for cyclic input of plotted data
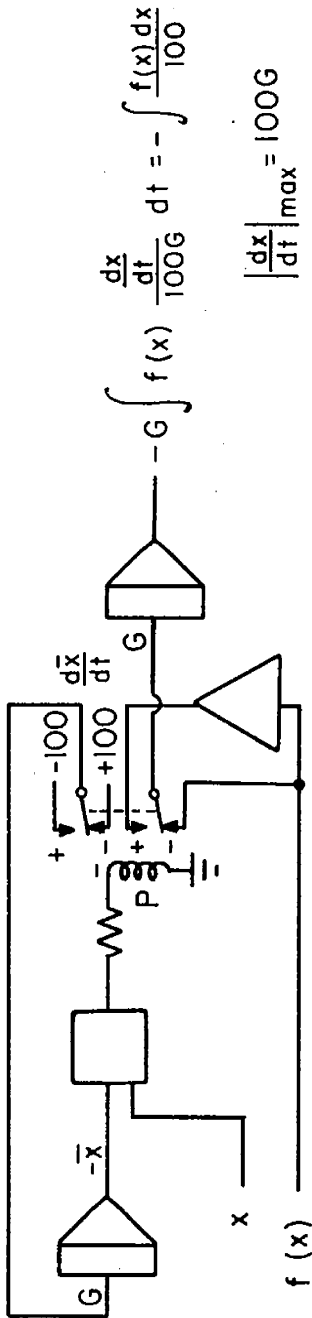
P = Polarized relay

Fig. 66 — Generalized integrator circuit
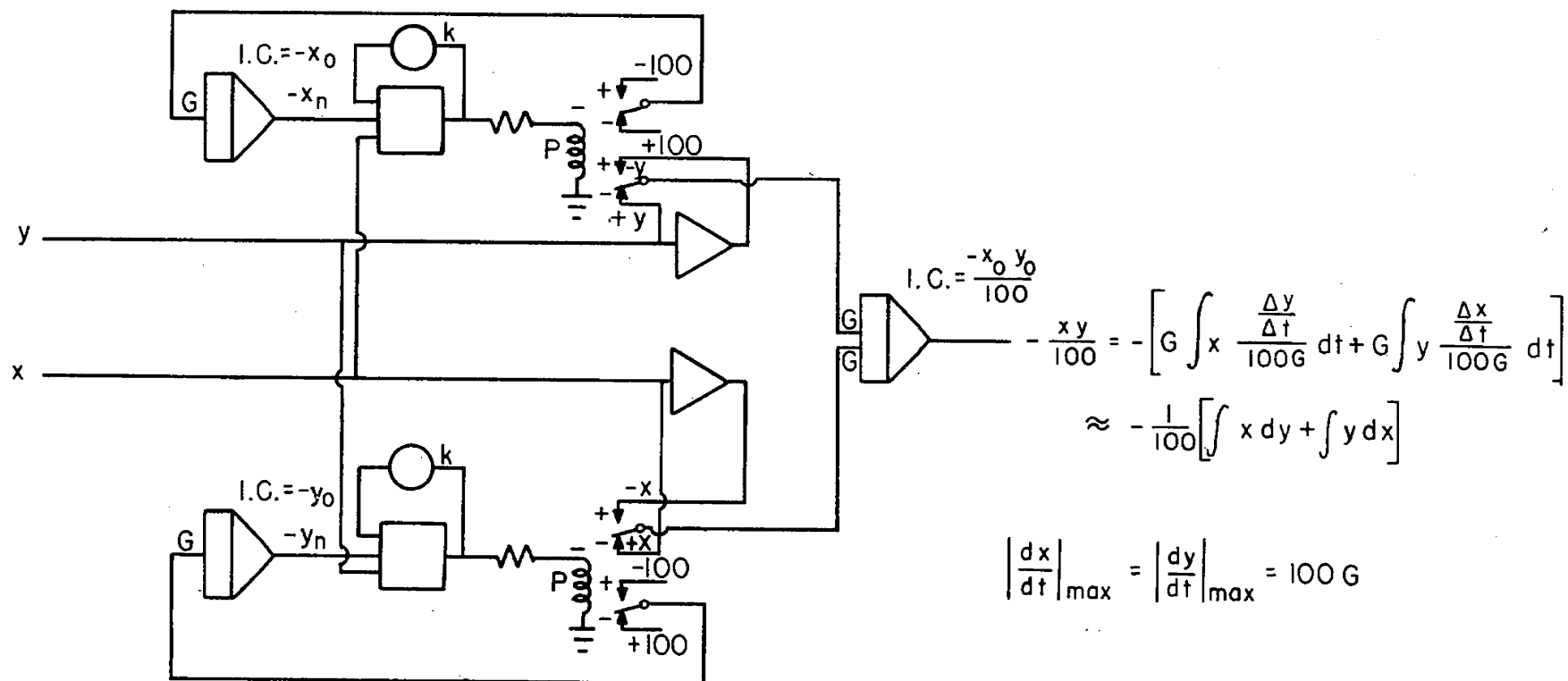
P = Polarized relays

Fig. 67 —— Relay multiplying circuit

I. Indeterminate Functions

Indeterminate functions have reared their nasty heads
several times in our work and until recently no satisfactory method
of handling them was available. The following methods were suggested
by Dr. Brown; the first one seems most satisfactory for any problems
we have encountered and requires the minimum of additional equipment.
The only disadvantage of the method is the requirement of analysis
prior to REAC computation.

Assume we wish to compute $F(t) = \frac{f(t)}{g(t)}$ where $f(0) = g(0) = 0$
but $F(0)$ is finite. It is usually possible to form a good
approximation of F by another function G in the neighborhood of
$t = 0$. We then can approximate F continuously by the linear
combination $F \approx \frac{\lambda(t)G + g(t)F}{\lambda(t) + g(t)} = \frac{\lambda(t)G + f(t)}{\lambda(t) + g(t)}$. The function $\lambda(t)$
is chosen such that it dies out after $g(t)$ becomes large enough to
permit accurate division but before G becomes a poor approximation.
An exponential seems satisfactory for $\lambda(t)$ since it is simple to
generate and satisfies the above requirements. The approximation
then becomes

$$F \approx \frac{be^{-at}G + f(t)}{be^{-at} + g(t)}.$$

For example, in computing $\frac{\sin t}{t}$, $G = 1$ and

$$F \approx \frac{be^{-at} + \sin t}{be^{-at} + t}.$$

Adjustment of a and b permits varying the system to satisfy various
G's and accuracy demands. A few trial runs of a given problem will
usually indicate if the particular $G(t)$ chosen is a suitable
approximation.

A second method that has been used demands more machine
components. An analysis of F will usually permit determination of
upper and lower bounds of the function for small values of t. For

example, an alternating series approximation gives upper or lower bounds as terms are added. If all terms of the series are positive, the value given after any term has been added is always a lower bound, and an arbitrary additive function can be chosen giving the upper bound. Division can then be done as usual except that the quotient will be limited by the upper and lower bounds determined as above. The course of $F(t)$ in the first few runs will indicate how well the bounds were chosen.

If the indeterminate function is of the form of a continuous average,

$$F(x) = \frac{1}{x} \int_0^x f(t)dt,$$

and $f(t)$ is monotonic, the mean value theorem provides a method of solution. We then need to find γ where $F(x) = f(\gamma x)$, and $0 < \gamma < 1$. A high-gain amplifier can be used to compute γ and has as an input

$$\left(x \cdot f(\gamma x) - \int_0^x f(t)dt \right)$$

It is true that γ will not be well determined initially, but at that time x, and hence the product γx, will be zero or very small, and we obtain $F(0) = f(0)$ as desired.

The solution of Bessel's Equation of higher order is a good example illustrating the need for the above techniques. Here we have

$$\ddot{y} = \frac{- t\dot{y} - (t^2 - n^2)y}{t^2}$$

Not only is a division by t^2 necessary, but for n's greater than one there is no driving voltage since $y(0) = \dot{y}(0) = 0$. While it is true that drift and noise voltages in the system will eventually force a "solution" to be obtained, there is no way of determining

the scale factor of the output or the location of $t = 0$. The first method of approximation was particularly applicable to this problem. The first term of the series expansion proved to be a suitable approximation of $\ddot{y}$ for small t. The resulting equation is

$$\ddot{y} = \frac{\left(\dfrac{t^{n-2}}{2^n(n-2)!}\right) be^{-at} - t\dot{y} - (t^2 - n^2)y}{be^{-at} + t^2}.$$

J. Potentiometer Loading Compensation

Since it is usually impossible to connect the arm of one potentiometer to another potentiometer, a large number of amplifiers are often required purely for isolation. However, if the loading is properly taken into account, potentiometer to potentiometer connections are quite feasible. These techniques should be used only where problem complication has made computing amplifiers unavailable as some loss of accuracy is to be expected in most cases.

For example, it is possible to load multiplying potentiometers on a servo multiplier if the follow-up potentiometer is identically loaded. This scheme depends on the total resistance of multiplying and follow-up potentiometers being equal. Actually the REAC potentiometers have a ± 2 per cent variation in their total resistance. However, as the following well-known analysis shows, variations in total potentiometer resistance have a second-order effect.

The apparent setting of a potentiometer loaded by another potentiometer (Figure 68) is given by:

$$K = \frac{k\,\rho}{k - k^2 + \rho}, \text{ where}$$

K = apparent setting,

k = setting of first potentiometer,

ρ = ratio of resistances.

Since $\dfrac{dK}{K} = \left(1 - \dfrac{K}{k}\right)\dfrac{d\rho}{\rho}$, and $\dfrac{d}{dK}\left(\dfrac{K}{k}\right) = \dfrac{(2k - 1)\rho}{(\rho + k^2 - k)^2}$, then, for $\rho \approx 1$,

$$(K/k)_{min} \approx 0.8 \text{ at } k = \frac{1}{2}.$$

It follows that

$$\left(\frac{dK}{K}\right)_{max} = \frac{1}{5}\frac{d\rho}{\rho}.$$
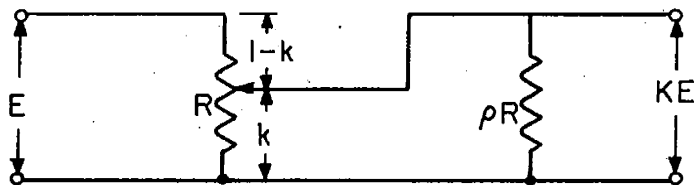
Fig. 68 —— Potentiometer loading

f(x) = true value of function

$\overline{f}(x)$ = compensated value
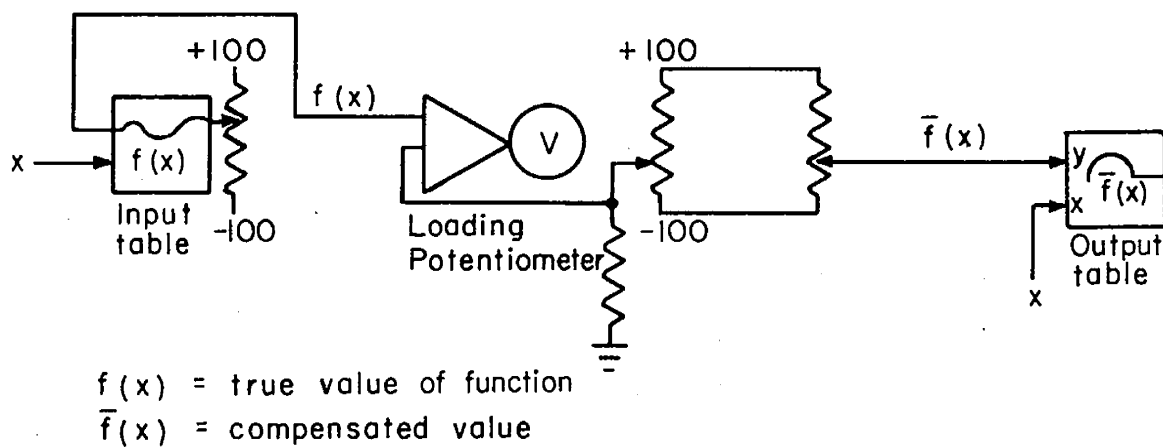
Fig. 69 —— REAC correction of input data to permit potentiometer loading

Hence a resistance difference of one per cent would result in a maximum loading compensation error of 0.2 per cent. If care is taken in matching potentiometers, reasonable accuracy will result.

It is possible to make a direct connection from an input table slide wire to a potentiometer if the curve is redrawn prior to computation to take loading into account. Figure 69 shows how the REAC can be utilized to perform the transformation for a function of one variable. The function $\overline{f}(x)$ is taken from the output table and used in place of $f(x)$ on an input table in the solution of the main problem. A more complicated circuit is required for a function of two variables but the saving in amplifiers is greater.

K. Limiting an Integral

The simple diode limiter is not sufficient for limiting the output of an integrator, for the derivative must be brought to zero whenever the integral is limited. Otherwise a change of sign of the derivative will not cause the limited integral to change until the unlimited output of the integrator returns to the limited value.

The circuit of Figure 70 is one way of limiting an integral. When x is within its limits, $k(x - x_{lim}) = 0$ and the feedback loop has no effect. Whenever x exceeds its limits the feedback loop forces x to remain at the limited value x_{lim}. This circuit also sharpens the cutoff characteristics of the diodes in the limiters. See Appendix II for a method of limiting using a diode in the feedback loop of the integrator.
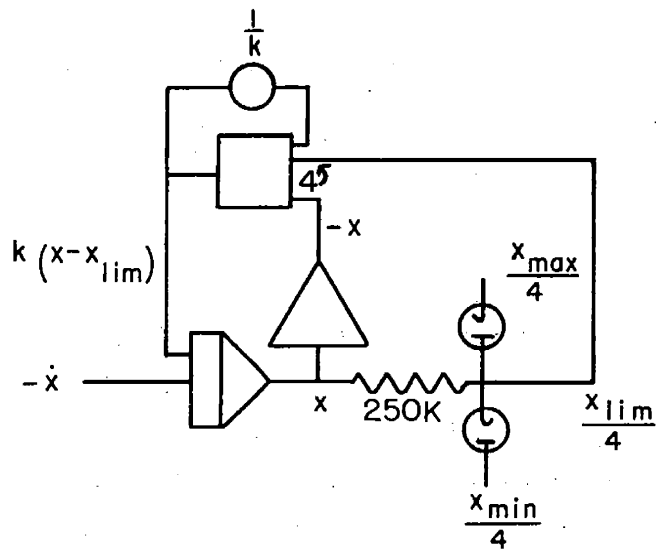
Fig. 70 — Circuit for limiting an integral

L. Deadspace and Backlash

Physical systems often
require functions of the type
shown to the right (deadspace).
The top circuit under the graph is
probably the simplest for gener-
ating this function, but the lower
circuit will eliminate possible
troubles caused by potentiometer
loading of amplifier 1.
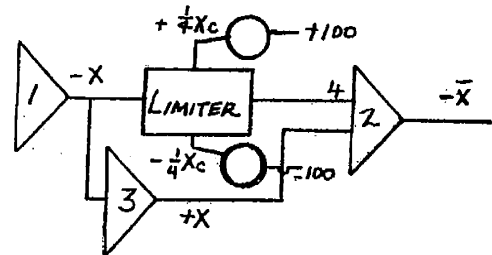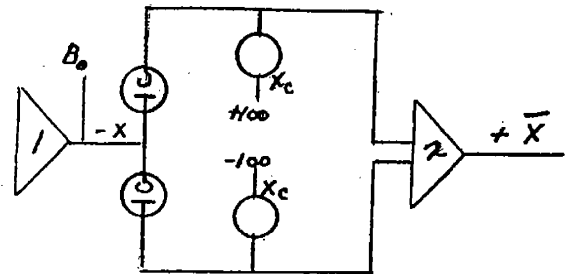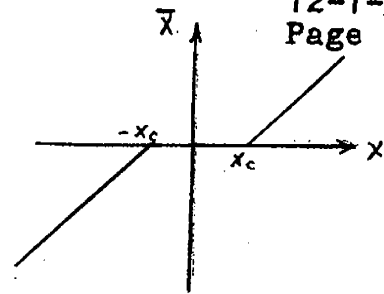
Deadspace

A similar physical system
is backlash (or hysteresis) in which
the variable $\bar{x}$ lags $\triangle x$ behind x as
shown at the right. The circuit
below the graph is one way of
generating this function. Notice
that $\dot{x}$ is required, but this usually
creates no problem, since if x is
changing, $\dot{x}$ is generally available
elsewhere in the circuit.

$\dot{\bar{X}} = 0$ WHEN $|\bar{x}-x| < \triangle x$

$\dot{\bar{X}} = \dot{X}$ WHEN $|\bar{x}-x| = \triangle x$
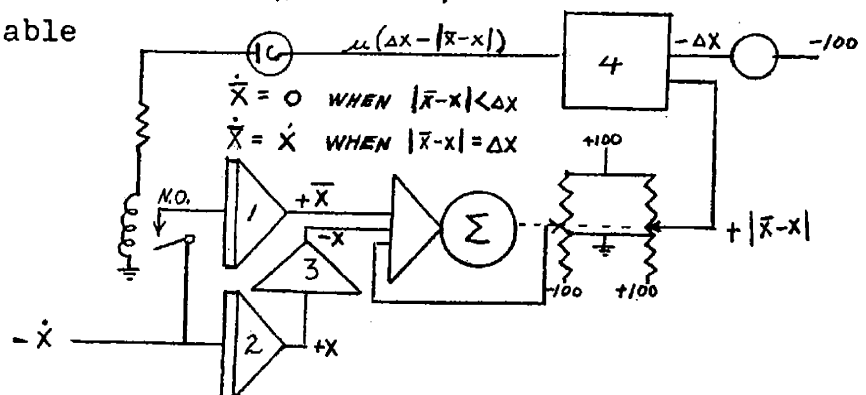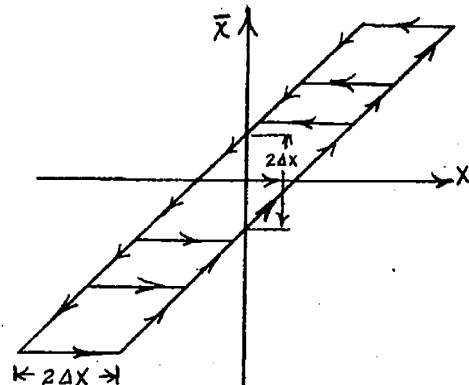
Backlash

M. Accuracy

The need for an estimate of the maximum errors or some sort of confidence limits for every problem done on the REAC cannot be overemphasized. A minimum sort of machine check consists in putting reasonable initial conditions across all integrators and checking the output of all amplifiers, servo-multipliers, and input tables against computed values. The duplication of a hand-computed run requires more preparation but gives greater assurance of accurate results. Checks of these types are not entirely satisfactory, however, since other runs may be more sensitive to REAC errors than the trial run, and consequently would not yield as accurate results as predicted.

Several types of minor checks are possible, usually depending on the type of problem. Mathematical definitions sometimes permit a continuous check on certain aspects of a problem. A set of simultaneous equations is usually best checked by studying one equation at a time with the inputs to the integrators for the other equations opened. Reversing the sign of all input and initial condition voltages and rerunning will determine whether or not drift or unbalanced power supplies are having deleterious effects. Doubling the computing time by doubling the number of condensers across all integrators and rerunning will indicate whether or not frequency limits are causing errors.

A good deal of confidence could be placed in the results of the following problem because of the many satisfactory check runs made.

$$\psi(R) = 1 - \left(\frac{1}{2}\right)^{\left(\frac{R_0}{R}\right)^4}$$

$$, \quad R = a + bt + ct^2$$

$$9 \frac{dx_1}{dt} = p\psi x_2 - x_1$$

$$9 \frac{dx_2}{dt} = (1 - \psi)x_1 - p\psi x_2$$

$$9 \frac{dx_3}{dt} = \psi x_1$$

$$P(R_0) = \frac{1}{18} \cdot 1.45\left(\frac{R_0}{18}\right)^{.45}$$

$$x_{1_0} = x_{3_0} = 0; \quad x_{2_0} = 1.0.$$

Plot

$$\bar{x}_3(t) = \int_0^{18} x_3(t, R_0) P(R_0) dR_0$$

The solution would be straightforward were it not for the last equation which causes the problem to have two independent variables. The REAC solution approximated the final equation by a four-point numerical integration formula, taking into account the nature of the function $P(R_0)$, which is zero but has an infinite derivative at $R_0 = 0$. Three sets of the differential equations were solved simultaneous and summed with the proper weighting functions to yield $\bar{x}_3(t)$ in one run. To eliminate the need for taking a square-root three ψ curves were plotted as functions of R^2 and entered from automatic input tables. Six of the fifty solutions had to be re-run using a five-point numerical integration formula to eliminate errors caused by the discrete nature of the solution.

Holding the three ψ curves constant gave results that could be checked analytically. All check points indicated an error of less than 0.2 per cent of full scale and the three sets of REAC solutions agreed with each other within 0.1 per cent of full scale. Since

$$\sum_{i=1}^{3} \frac{dx_i}{dt} = \frac{d}{dt} \sum_{i=1}^{3} x_i = 0,$$

the sum of the x's should be constant at all times and was checked to be so within 0.1 per cent. A numerical check was made on the

generation of R by the polynomial in t once during each run. The
input functions plotted on the output table within 0.2 per cent
of their true value. Doubling the solution time had no effect on
the output.

A numerical check or improvement of REAC solutions either
by hand or IBM equipment is advisable in most cases. The most
straightforward approach involves application of the method used
in the fundamental existence theorem proof based on Picard's
classical process of successive approximations. Most equations may
be reduced to the form

$$\dot{x}_i = f_i(x_1,x_2,\ldots,x_n,t), \quad i = 1,2,\ldots,n.$$

Plots are made of all x_i's as a function of time, permitting
numerical computation of the $\dot{x}_i$ as functions of time. Numerical
integration then yields a better estimate of the x_i. This method
may be repeated until the desired accuracy is achieved, and is useful
in improving solutions of problems so complicated as to require
approximations to fit on the REAC.

The following numerical integration formulae are useful
for the above application:

$$x_1 = x_0 + \frac{h}{12}(5\dot{x}_0 + 8\dot{x}_1 - \dot{x}_2)$$

$$x_2 = x_0 + \frac{h}{3}(\dot{x}_0 + 4\dot{x}_1 + \dot{x}_2)$$

$$x_2 = x_1 + \frac{h}{24}(-\dot{x}_0 + 13\dot{x}_1 + 13\dot{x}_2 - \dot{x}_3)$$

$$x_f = x_{f-1} + \frac{h}{12}(-\dot{x}_{f-2} + 8\dot{x}_{f-1} + 5\dot{x}_f)$$

where in this case $x_0,x_1,x_2,\ldots$ are the values of the function
at equally spaced points $t_0,t_1,t_2,\ldots$ with interval h.

IV. PLUGBOARD WIRING

Flexibility and convenience were the objectives of the design of
the plugboard. Flexibility was achieved by bringing out to the
plugboard all wiring connections which can be altered in the course
of constructing an analogue from the components of the computer;
convenience, by arranging the plugboard connections so that the
most frequently used connections are easily made.

The plugboard connections of the major components are divided into
two groups according to the function they serve. One group, which
occupies the two center panels of the plugboard, contains the
input and output connections of the major components. This group
corresponds to the connections on the jacks of the original REAC
patch bay. The other group, located in the first and fourth panels
of the plugboard contains the wiring connections which are concerned
with the control of the operation of the components. Minor
components such as relays, switches, extra condensers, etc., are
located wherever unused terminals were conveniently available on
the plugboard. The connections in the control section of the
plugboard are so arranged that all the wiring required for a set
of frequently used operations, which might be called standard
operations, can be accomplished with "bottle" plugs. Much of the
control wiring will be the same for all problems, and because it
is separate from the input output wiring, it is convenient to leave
it in the plugboards. The technique of arranging the connections
to make wiring easy is used in the input-output section too. Outputs
from D.C. amplifiers are spread to rows located adjacent to inputs
to servo amplifiers, to inputs, to scale factor pots, and to inputs of
the servo multiplying pots. Outputs from pot arms are in rows
adjacent to both the D.C. amplifier and servo amplifier inputs.

In order to specify plugboard wiring some system of labeling must
be used to designate the plugholes. One system is that used on the
plugboard. (See Plugboard Diagram) Labels written above groups of
holes indicate their function. Columns are numbered to indicate the
number of the component with which individual plugholes are connected.
As wires are plugged into the board it becomes increasingly difficult
to read the labels, and familiarity with the plugboard layout is
required in order to locate specified connections to components
quickly. This system of designation by function and component number
has the further disadvantage that multiple connections, such as
D.C. amplifier outputs, are not uniquely specified. The lack of
uniqueness seriously hampers checking of the wiring in the plugboard.

A rectangular coordinate system designating plugholes by the row
and column in which they are located avoids the difficulties
mentioned above. In the table and diagrams which follow a rectangular

system is set up by numbering the columns 1 to 80 from left to
right across the front of the plugboard, and the rows 1 to 34
from top to bottom of the plugboard. Designations are written
row/column or (row, column). Figures 71 to 76 are block diagrams
of comonents showing connections which are brought out to the
plugboard and giving a simple formula for using the component
number to determine the row and column location of each connection.
Special cases for which no simple formula could be written are
labeled i, j, k, or ℓ. Table VII lists by components all connections
which are made to the plugboard. The block diagrams will probably
be most useful when specifying the wiring for a circuit; however,
Table VII or the Plugboard Diagram may be required occasionally for
reference. The Plugboard Diagrams for the four panels appear in
the next four pages.

Panel 1

RM525-4

Panel   2

Panel 3

Panel 4

Row	3	4	5	6	7	8			11	12	13	19	23	27
Column			n + 20								n + 20			

DC Amplifier inputs

DC Amplifier outputs

All 1 meg

100k   100k

D.C. Amplifier

Feedback impedance

+300v

IC   Summing junction   Grid   Feedback   Hold relay   Reset relay   Output   Boost

Row		20		21				i	22		23		24		26		k	28		29
Column		$\bar{n}$		$\bar{n}$				j	$\bar{n}$		$\bar{n}$		$\bar{n}$		$\bar{n}$		ℓ	$\bar{n}$		$\bar{n}$

$n \leq 20$ read $\bar{n} = n$
$n > 20$ read $\bar{n} = n + 40$ and omit $20/\bar{n}$, $28/\bar{n}$, $29/\bar{n}$, i/j and k/ℓ

100k

Balance

100k

IC

100k

Reset

Integrate

Hold

Fig. 71 — D.C. Amplifier

[Servos 5, 6, 7, 8, 13, 14, 15, and 16
see fig. 76 resolving servos]

Row	34	33		32		31		30		i	14	15	16	17		14	15	16		28	29	31	32		33
Column	n	n		n		n		n		j		n + 20					n + 40					n + 60			

Sec   FB   F   Sum   Servo   Follow-up   Follow-up   Pot
Input   Res   Arm Junction   Inputs   Pot arm   Hi   Lo   Ct

Fig. 72 — Multiplying servo

Row		1	20	22	1			2	24	26	3		1	20	22	5			2	24	26			2	24	26
Column			n + 20		n+60				n + 20		n+60			n + 40		n+60				i + 40					i + 41	

Arm Hi Lo Ct   Arm Hi Lo Ct   Arm Hi Lo Ct   Arm Hi Lo   Arm Hi Lo
A Pot   B Pot   C Pot   R servo Sw   L servo Sw

Row	34	33		14	15	16	17		30		32		31			28	29	31	32		33
Col	n+16			n + 36					n+16		n+16		n+16			n + 76					

Secondary Input   I T Inputs   Summing Junction   Feedback Resistor   Follow-up Pot arm   Follow-up Pot Hi Lo   Follow-up Pot C.T.

+100v   -100v

2M 2M 1M 1M   2M   1M 1M

Push-pull Amplifier

Damping control

Critical damping switch

5k   0.5μf   100k   Servo Motor   A.C. Line

Damping Tachometer

Follow-up Pot

Hi   Lo   I II III   Hi Lo Arm

Slide   Wire   S.W. Arms   Pot

Row	20		22		1	2	1		24	26		2
Column	n+36		n+36		n+36		n+56		n+36	n+56		

Fig. 73 — Input table

Row	34	33		32		31		2	14	15	16	17			2	4	6
IT 5 Col		21						17		57						77	
OT Col		23						19		59						79	

Row	Col
20	57
22	57
2	52
2	53
2	54
2	55
2	56
20	58
22	58

Row	34	33		32		31		2	14	15	16	17			2	4	6
IT 5 Col		22						18		58						78	
OT Col		24						20		60						80	
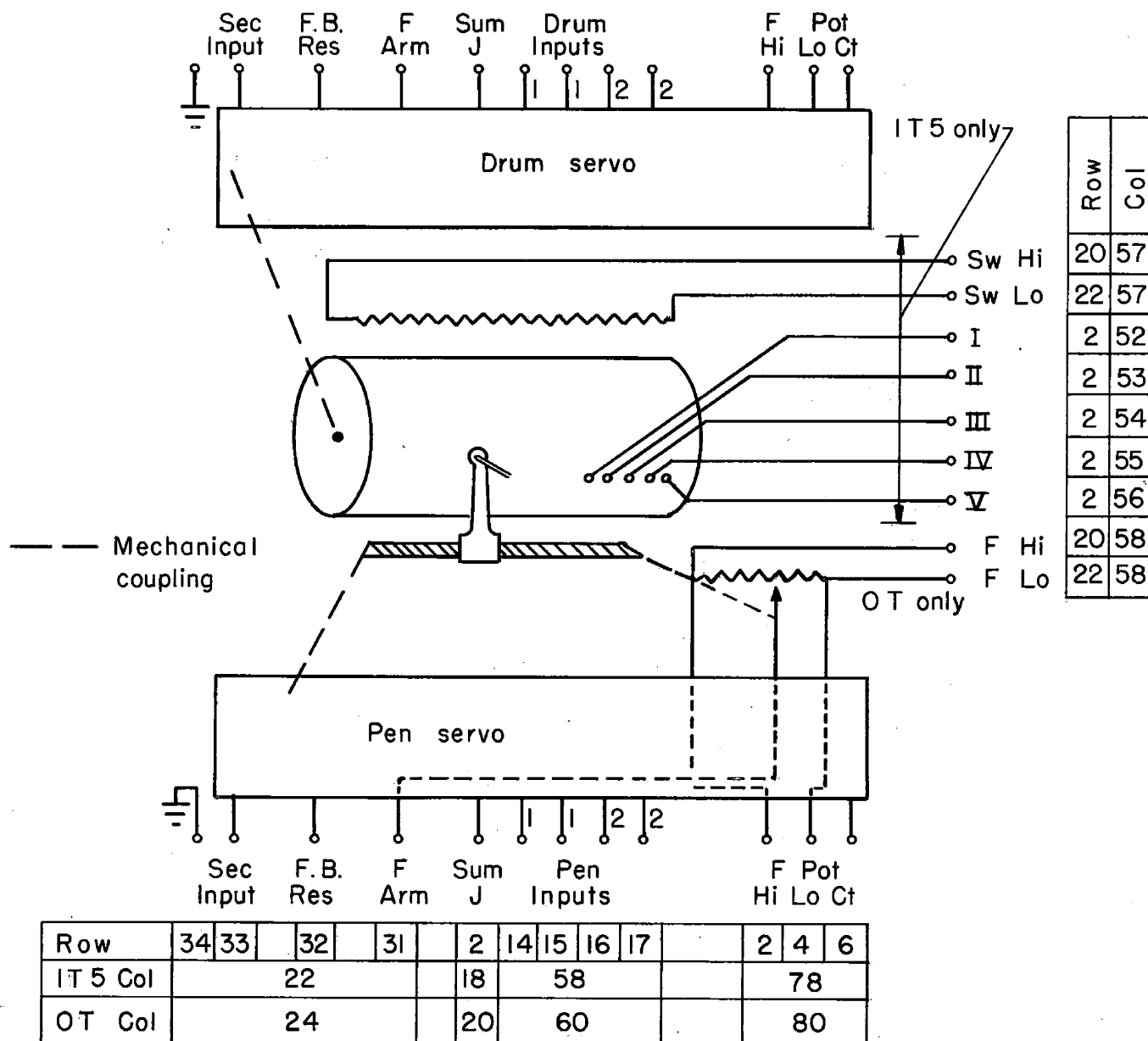
Fig. 74——I T 5 and output table

Row	10	29		9	18
Col			n + 20		

S. F. Pot

17		19		18
		n		

I. C. Pot

12		14		13
20		20		20

X T Pot

		16		
4n+57	4n+58	4n+59	4n+60	

Limiter

Hi   Arm

20 k

40 S. F. Pots

Hi   Arm   Lo

20 k

-100v +100v

− ⟷ +

6 I.C. Pots

Hi   Arm   Lo

40 Turns

I External Pot

In   −   +   Out

750 k

4 Limiters

Row		3 Relays rows 17, 18, and 19										
Column	9		10	11	12	13	14		15	16	17		18	19	20

Row		4 Relays rows 17, 18, 19, and 20										
Column	72		71	70	61	62	63		64	65	66		67	68	69

No A Nc   No A Nc   No A Nc

7 Relays

16		16
77		78

16		16
79		80

2 Diodes

Row		Rows 31, 32 and 33					
Col	25		26		27		28	29	30
Col	31		32		33		34	35	36

+   −   + Arm −

6 Differential relays

i	(3 to 11)	i
j	13 to 20	j+1

− Meg ohms

36 Resistors

i	i	(1 to 15)	i+1	i+1
j	j+1	(1 to 12)	j	j+1

− Microfarads

48 Condensers

Fig. 75—S. F. Pots and minor components

Note multiple connection to first input to servus i + 2 and i + 3
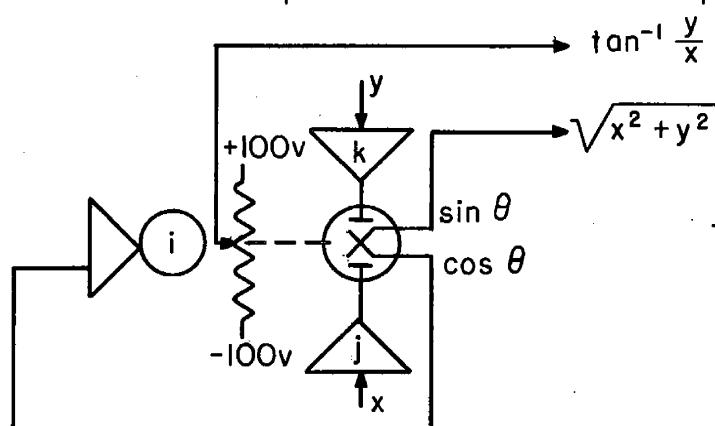
Polar operation
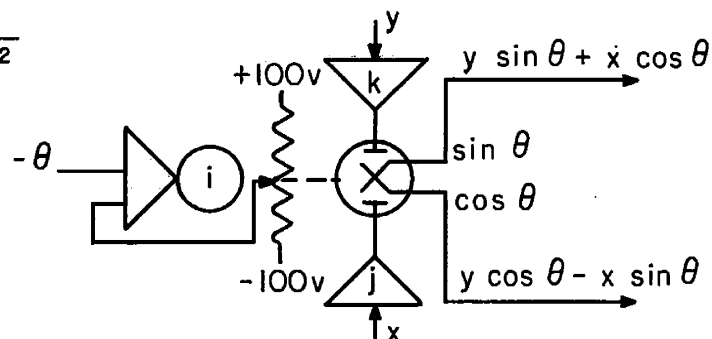
Rectangular operation

	θ				x					y					$\sin\theta$	$\cos\theta$
Servo 3, 5 and 6	14	15	16	17	17	14	15	16	17	17	14	15	16	17	17	17
	23	23	23	23	41	25	25	25	25	42	26	26	26	26	43	44
Servo 4, 7 and 8	14	15	16	17	17	14	15	16	17	17	14	15	16	17	17	17
	24	24	24	24	45	27	27	27	27	46	28	28	28	28	47	48
Servo 11, 13 and 14	14	15	16	17	17	14	15	16	17	17	14	15	16	17	17	17
	31	31	31	31	49	33	33	33	33	50	34	34	34	34	51	52
Servo 12, 15 and 16	14	15	16	17	17	14	15	16	17	17	14	15	16	17	17	17
	32	32	32	32	53	35	35	35	35	54	36	36	36	36	55	56

Fig. 76 — Resolving servos

TABLE VII. PLUGBOARD OPERATION

$\frac{a}{b} = \frac{row}{column}$	I.C. INPUT	SUM JUNCTION	HOLD RELAY	RESET RELAY	GRID	FEEDBACK IMPEDANCE	INPUTS	OUTPUTS
D.C. Amplifier 1	20 01	21 01	24 01	26 01	22 01 01 06	1 µf 23 01	03 04 05 06 07 08 21 21 21 21 21 21	11 12 13 19 23 27 28 02 21 21 21 21 21 21 01 06
D.C. Amplifier 2	20 02	21 02	24 02	26 02	22 04 02 06	1 µf 23 02	03 04 05 06 07 08 22 22 22 22 22 22	11 12 13 19 23 27 28 03 22 22 22 22 22 22 02 06
D.C. Amplifier 3	20 03	21 03	24 03	26 03	22 05 03 06	1 µf 23 03	03 04 05 06 07 08 23 23 23 23 23 23	11 12 13 19 23 27 28 06 23 23 23 23 23 23 03 06
D.C. Amplifier 4	20 04	21 04	24 04	26 04	22 08 04 06	1 µf 23 04	03 04 05 06 07 08 24 24 24 24 24 24	11 12 13 19 23 27 28 07 24 24 24 24 24 24 04 06
D.C. Amplifier 5	20 05	21 05	24 05	26 05	22 09 05 06	1 µf 23 05	03 04 05 06 07 08 25 25 25 25 25 25	11 12 13 19 23 27 28 10 25 25 25 25 25 25 05 06
D.C. Amplifier 6	20 06	21 06	24 06	26 06	22 12 06 06	1 µf 23 06	03 04 05 06 07 08 26 26 26 26 26 26	11 12 13 19 23 27 28 11 26 26 26 26 26 26 06 06
D.C. Amplifier 7	20 07	21 07	24 07	26 07	22 13 07 06	1 µf 23 07	03 04 05 06 07 08 27 27 27 27 27 27	11 12 13 19 23 27 28 14 27 27 27 27 27 27 07 06
D.C. Amplifier 8	20 08	21 08	24 08	26 08	22 16 08 06	1 µf 23 08	03 04 05 06 07 08 28 28 28 28 28 28	11 12 13 19 23 27 28 15 28 28 28 28 28 28 08 06
D.C. Amplifier 9	20 09	21 09	24 09	26 09	22 09	1 MΩ 23 09	03 04 05 06 07 08 29 29 29 29 29 29	11 12 13 19 23 27 28 01 29 29 29 29 29 29 09 13
D.C. Amplifier 10	20 10	21 10	24 10	26 10	22 10	1 MΩ 23 10	03 04 05 06 07 08 30 30 30 30 30 30	11 12 13 19 23 27 28 01 30 30 30 30 30 30 10 14

$\frac{\text{row}}{\text{column}}$ $\frac{a}{b} =$	I.C. INPUT	SUM JUNCTION	HOLD RELAY	RESET RELAY	GRID	FEEDBACK IMPEDANCE	INPUTS						OUTPUTS							
D.C. Amplifier 11	20 11	21 11	24 11	26 11	22 11	1 MΩ 23 11	03 31	04 31	05 31	06 31	07 31	08 31	11 31	12 31	13 31	19 31	23 31	27 31	28 11	01 15
D.C. Amplifier 12	20 12	21 12	24 12	26 12	22 12	1 MΩ 23 12	03 32	04 32	05 32	06 32	07 32	08 32	11 32	12 32	13 32	19 32	23 32	27 32	28 12	01 16
D.C. Amplifier 13	20 13	21 13	24 13	26 13	22 13	1 MΩ 23 13	03 33	04 33	05 33	06 33	07 33	08 33	11 33	12 33	13 33	19 33	23 33	27 33	28 13	01 17
D.C. Amplifier 14	20 14	21 14	24 14	26 14	22 14	1 MΩ 23 14	03 34	04 34	05 34	06 34	07 34	08 34	11 34	12 34	13 34	19 34	23 34	27 34	28 14	01 18
D.C. Amplifier 15	20 15	21 15	24 15	26 15	22 15	1 MΩ 23 15	03 35	04 35	05 35	06 35	07 35	08 35	11 35	12 35	13 35	19 35	23 35	27 35	28 15	01 19
D.C. Amplifier 16	20 16	21 16	24 16	26 16	22 16	1 MΩ 23 16	03 36	04 36	05 36	06 36	07 36	08 36	11 36	12 36	13 36	19 36	23 36	27 36	28 16	01 20
D.C. Amplifier 17	20 17	21 17	24 17	26 17	22 17	1 MΩ 23 17	03 37	04 37	05 37	06 37	07 37	08 37	11 37	12 37	13 37	19 37	23 37	27 37	28 17	02 13
D.C. Amplifier 18	20 18	21 18	24 18	26 18	22 18	1 MΩ 23 18	03 38	04 38	05 38	06 38	07 38	08 38	11 38	12 38	13 38	19 38	23 38	27 38	28 18	02 14
D.C. Amplifier 19	20 19	21 19	24 19	26 19	22 19	1 MΩ 23 19	03 39	04 39	05 39	06 39	07 39	08 39	11 39	12 39	13 39	19 39	23 39	27 39	28 19	02 15
D.C. Amplifier 20	20 20	21 20	24 20	26 20	22 20	1 MΩ 23 20	03 40	04 40	05 40	06 40	07 40	08 40	11 40	12 40	13 40	19 40	23 40	27 40	28 20	02 16

$\frac{q}{b} = \frac{row}{column}$	I.C. INPUT	SUM JUNCTION	HOLD RELAY	RESET RELAY	GRID	FEEDBACK IMPEDANCE	INPUTS						OUTPUTS					
D.C. Amplifier 21		21 61	24 61	26 61	22 61	1 MΩ 23 61	03 41	04 41	05 41	06 41	07 41	08 41	11 41	12 41	13 41	19 41	23 41	27 41
D.C. Amplifier 22		21 62	24 62	26 62	22 62	1 MΩ 23 62	03 42	04 42	05 42	06 42	07 42	08 42	11 42	12 42	13 42	19 42	23 42	27 42
D.C. Amplifier 23		21 63	24 63	26 63	22 63	1 MΩ 23 63	03 43	04 43	05 43	06 43	07 43	08 43	11 43	12 43	13 43	19 43	23 43	27 43
D.C. Amplifier 24		21 64	24 64	26 64	22 64	1 MΩ 23 64	03 44	04 44	05 44	06 44	07 44	08 44	11 44	12 44	13 44	19 44	23 44	27 44
D.C. Amplifier 25		21 65	24 65	26 65	22 65	1 MΩ 23 65	03 45	04 45	05 45	06 45	07 45	08 45	11 45	12 45	13 45	19 45	23 45	27 45
D.C. Amplifier 26		21 66	24 66	26 66	22 66	1 MΩ 23 66	03 46	04 46	05 46	06 46	07 46	08 46	11 46	12 46	13 46	19 46	23 46	27 46
D.C. Amplifier 27		21 67	24 67	26 67	22 67	1 MΩ 23 67	03 47	04 47	05 47	06 47	07 47	08 47	11 47	12 47	13 47	19 47	23 47	27 47
D.C. Amplifier 28		21 68	24 68	26 68	22 68	1 MΩ 23 68	03 48	04 48	05 48	06 48	07 48	08 48	11 48	12 48	13 48	19 48	23 48	27 48
D.C. Amplifier 29		21 69	24 69	26 69	22 69	1 MΩ 23 69	03 49	04 49	05 49	06 49	07 49	08 49	11 49	12 49	13 49	19 49	23 49	27 49
D.C. Amplifier 30		21 70	24 70	26 70	22 70	1 MΩ 23 70	03 50	04 50	05 50	06 50	07 50	08 50	11 50	12 50	13 50	19 50	23 50	27 50

row/column a=b	I.C. INPUT	SUM JUNCTION	HOLD RELAY	RESET RELAY	GRID	FEEDBACK IMPEDANCE	INPUTS	OUTPUTS
D.C. Amplifier 31		21 / 71	24 / 71	26 / 71	22 / 71	1 MΩ 23 / 71	03 04 05 06 07 08 / 51 51 51 51 51 51	11 12 13 19 23 27 / 51 51 51 51 51 51
D.C. Amplifier 32		21 / 72	24 / 72	26 / 72	22 / 72	1 MΩ 23 / 72	03 04 05 06 07 08 / 52 52 52 52 52 52	11 12 13 19 23 27 / 52 52 52 52 52 52
D.C. Amplifier 33		21 / 73	24 / 73	26 / 73	22 / 73	1 MΩ 23 / 73	03 04 05 06 07 08 / 53 53 53 53 53 53	11 12 13 19 23 27 / 53 53 53 53 53 53
D.C. Amplifier 34		21 / 74	24 / 74	26 / 74	22 / 74	1 MΩ 23 / 74	03 04 05 06 07 08 / 54 54 54 54 54 54	11 12 13 19 23 27 / 54 54 54 54 54 54
D.C. Amplifier 35		21 / 75	24 / 75	26 / 75	22 / 75	1 MΩ 23 / 75	03 04 05 06 07 08 / 55 55 55 55 55 55	11 12 13 19 23 27 / 55 55 55 55 55 55
D.C. Amplifier 36		21 / 76	24 / 76	26 / 76	22 / 76	1 MΩ 23 / 76	03 04 05 06 07 08 / 56 56 56 56 56 56	11 12 13 19 23 27 / 56 56 56 56 56 56
D.C. Amplifier 37		21 / 77	24 / 77	26 / 77	22 / 77	1 MΩ 23 / 77	03 04 05 06 07 08 / 57 57 57 57 57 57	11 12 13 19 23 27 / 57 57 57 57 57 57
D.C. Amplifier 38		21 / 78	24 / 78	26 / 78	22 / 78	1 MΩ 23 / 78	03 04 05 06 07 08 / 58 58 58 58 58 58	11 12 13 19 23 27 / 58 58 58 58 58 58
D.C. Amplifier 39		21 / 79	24 / 79	26 / 79	22 / 79	1 MΩ 23 / 79	03 04 05 06 07 08 / 59 59 59 59 59 59	11 12 13 19 23 27 / 59 59 59 59 59 59
D.C. Amplifier 40		21 / 80	24 / 80	26 / 80	22 / 80	1 MΩ 23 / 80	03 04 05 06 07 08 / 60 60 60 60 60 60	11 12 13 19 23 27 / 60 60 60 60 60 60

a = row / b = column	SECONDARY INPUT	SUMMING JUNCTION	FEEDBACK RESISTOR	FOLLOW-UP POT. ARM	FOLLOW-UP POT. HIGH LOW C.T.	SERVO INPUTS	A POT ARM HIGH LOW C.T.	B POT ARM HIGH LOW C.T.	C POT ARM HIGH LOW C.T.	L ARM / L HIGH SLIDE- / L LOW WIRE SERVO	R ARM / R HIGH SLIDE- / R LOW WIRE SERVO	SIN	SOC
Servo 1	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05	02 24 26	02 24 26		
	01	01	01	01 41 41 41	61 61 61	21 21 21 21	21 21 21 61	21 21 21 61	41 41 41 61	41 41 41	42 42 42		
Servo 2	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05	02 24 26	02 24 26		
	02	02	02	02 42 42 42	62 62 62	22 22 22 22	22 22 22 62	22 22 22 62	42 42 42 62	43 43 43	44 44 44		
Servo 3	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05			17	17
	03	03	03	03 43 43 43	63 63 63	23 23 23 23	23 23 23 63	23 23 23 63	43 43 43 63			43	44
Servo 4	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05			17	17
	04	04	04	04 44 44 44	64 64 64	24 24 24 24	24 24 24 64	24 24 24 64	44 44 44 64			47	48
Servo 5	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05				
	05	05	05	05 45 45 45	65 65 65	25 25 25 25	25 25 25 65	25 25 25 65	45 45 45 65				
X						$\frac{17}{41}$							
Servo 6	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05				
	06	06	06	06 46 46 46	66 66 66	26 26 26 26	26 26 26 66	26 26 26 66	46 46 46 66				
Y						$\frac{17}{42}$							
Servo 7	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05				
	07	07	07	07 47 47 47	67 67 67	27 27 27 27	27 27 27 67	27 27 27 67	47 47 47 67				
X						$\frac{17}{45}$							
Servo 8	33	30	32	31 14 15 16	29 31 33	14 15 16 17	01 20 22 01	02 24 26 03	01 20 22 05				
	08	08	08	08 48 48 48	68 68 68	28 28 28 28	28 28 28 68	28 28 28 68	48 48 48 68				
Y						$\frac{17}{46}$							

$\dfrac{a}{b} = \dfrac{\text{row}}{\text{column}}$	SECONDARY INPUT	SUMMING JUNCTION	FEEDBACK RESISTOR	FOLLOW-UP POT. ARM				HIGH FOLLOW-UP POT.			SERVO INPUTS				A POT ARM HIGH LOW C.T.				B POT ARM HIGH LOW C.T.				C POT ARM HIGH LOW C.T.				L ARM SERVO / L HIGH SLIDE / L LOW WIRE			R ARM SERVO / R HIGH SLIDE / R LOW WIRE			SIN	COS
Servo 9	33/09	30/09	32/09	31/09	14/49	15/49	16/49	29/69	31/69	33/69	14/29	15/29	16/29	17/29	01/29	20/29	22/29	01/69	02/29	24/29	26/29	03/69	01/49	20/49	22/49	05/69	02/45	24/45	26/45	02/46	24/46	26/46		
Servo 10	33/10	30/10	32/10	31/10	14/50	15/50	16/50	29/70	31/70	33/70	14/30	15/30	16/30	17/30	01/30	20/30	22/30	01/70	02/30	24/30	26/30	03/70	01/50	20/50	22/50	05/70	02/47	24/47	26/47	02/48	24/48	26/48		
Servo 11	33/11	30/11	32/11	31/11	14/51	15/51	16/51	29/71	31/71	33/71	14/31	15/31	16/31	17/31	01/31	20/31	22/31	01/71	02/31	24/31	26/31	03/71	01/51	20/51	22/51	05/71							17/51	17/52
Servo 12	33/12	30/12	32/12	31/12	14/52	15/52	16/52	29/72	31/72	33/72	14/32	15/32	16/32	17/32	01/32	20/32	22/32	01/72	02/32	24/32	26/32	03/72	01/52	20/52	22/52	05/72							17/55	17/56
Servo 13 X	33/13	30/13	32/13	31/13	14/53	15/53	16/53	29/73	31/73	33/73	14/33 15/33 16/33 17/33 17/49				01/33	20/33	22/33	01/73	02/33	24/33	26/33	03/73	01/53	20/53	22/53	05/73								
Servo 14 Y	33/14	30/14	32/14	31/14	14/54	15/54	16/54	29/74	31/74	33/74	14/34 15/34 16/34 17/34 17/50				01/34	20/34	22/34	01/74	02/34	24/34	26/34	03/74	01/54	20/54	22/54	05/74								
Servo 15 X	33/15	30/15	32/15	31/15	14/55	15/55	16/55	29/75	31/75	33/75	14/35 15/35 16/35 17/35 17/53				01/35	20/35	22/35	01/75	02/35	24/35	26/35	03/75	01/55	20/55	22/55	05/75								
Servo 16 Y	33/16	30/16	32/16	31/16	14/56	15/56	16/56	29/76	31/76	33/76	14/36 15/36 16/36 17/36 17/54				01/36	20/36	22/36	01/76	02/36	24/36	26/36	03/76	01/56	20/56	22/56	05/76								

$\dfrac{a}{b} = \dfrac{\text{row}}{\text{column}}$	SUMMING JUNCTION	FOLLOW-UP POT ARM	FEEDBACK RESISTOR	SECONDARY INPUT	SLIDE-WIRE ARM I	II	III	IV	V	INPUTS 1	1	2	2	SLIDE-WIRE HIGH	LOW	H POT HIGH	LOW	ARM	F POT HIGH	LOW	C.T.
Input Table 1	30	31	32	33	01	02	01			14	15	16	17	20	22	24	26	02	29	31	33
	17	17	17	17	37	37	57			37	37	37	37	37	37	37	37	57	77	77	77
Input Table 2	30	31	32	33	01	02	01			14	15	16	17	20	22	24	26	02	29	31	33
	18	18	18	18	38	38	58			38	38	38	38	38	38	38	38	58	78	78	78
Input Table 3	30	31	32	33	01	02	01			14	15	16	17	20	22	24	26	02	29	31	33
	19	19	19	19	39	39	59			39	39	39	39	39	39	39	39	59	79	79	79
Input Table 4	30	31	32	33	01	02	01			14	15	16	17	20	22	24	26	02	29	31	33
	20	20	20	20	40	40	60			40	40	40	40	40	40	40	40	60	80	80	80
Input Table 5 Drum	02	31	32	33						14	15	16	17						02	04	06
	17	21	21	21						57	57	57	57						77	77	77
Input Table 5 Pen	02	31	32	33	02	02	02	02	02	14	15	16	17	20	22				02	04	06
	18	22	22	22	52	53	54	55	56	58	58	58	58	57	57				78	78	78
Output Table Drum	02	31	32	33						14	15	16	17						02	04	06
	19	23	23	23						59	59	59	59						79	79	79
Output Table Pen	02	31	32	33						14	15	16	17						02	04	06
	20	24	24	24						60	60	60	60						80	80	80
																			20	22	
																			58	58	

a = row / b = column	S.F. POT HIGH	S.F. POT HIGH	S.F. POT ARM	S.F. POT ARM
S.F. Pot 1	10	29	09	18
	21	21	21	21
S.F. Pot 2	10	29	09	18
	22	22	22	22
S.F. Pot 3	10	29	09	18
	23	23	23	23
S.F. Pot 4	10	29	09	18
	24	24	24	24
S.F. Pot 5	10	29	09	18
	25	25	25	25
S.F. Pot 6	10	29	09	18
	26	26	26	26
S.F. Pot 7	10	29	09	18
	27	27	27	27
S.F. Pot 8	10	29	09	18
	28	28	28	28
S.F. Pot 9	10	29	09	18
	29	29	29	29
S.F. Pot 10	10	29	09	18
	30	30	30	30

	S.F. POT HIGH	S.F. POT HIGH	S.F. POT ARM	S.F. POT ARM
S.F. Pot 11	10	29	09	18
	31	31	31	31
S.F. Pot 12	10	29	09	18
	32	32	32	32
S.F. Pot 13	10	29	09	18
	33	33	33	33
S.F. Pot 14	10	29	09	18
	34	34	34	34
S.F. Pot 15	10	29	09	18
	35	35	35	35
S.F. Pot 16	10	29	09	18
	36	36	36	36
S.F. Pot 17	10	29	09	18
	37	37	37	37
S.F. Pot 18	10	29	09	18
	38	38	38	38
S.F. Pot 19	10	29	09	18
	39	39	39	39
S.F. Pot 20	10	29	09	18
	40	40	40	40

	S.F. POT HIGH	S.F. POT HIGH	S.F. POT ARM	S.F. POT ARM
S.F. Pot 21	10	29	09	18
	41	41	41	41
S.F. Pot 22	10	29	09	18
	42	42	42	42
S.F. Pot 23	10	29	09	18
	43	43	43	43
S.F. Pot 24	10	29	09	18
	44	44	44	44
S.F. Pot 25	10	29	09	18
	45	45	45	45
S.F. Pot 26	10	29	09	18
	46	46	46	46
S.F. Pot 27	10	29	09	18
	47	47	47	47
S.F. Pot 28	10	29	09	18
	48	48	48	48
S.F. Pot 29	10	29	09	18
	49	49	49	49
S.F. Pot 30	10	29	09	18
	50	50	50	50

	S.F. POT HIGH	S.F. POT HIGH	S.F. POT ARM	S.F. POT ARM
S.F. Pot 31	10	29	09	18
	51	51	51	51
S.F. Pot 32	10	29	09	18
	52	52	52	52
S.F. Pot 33	10	29	09	18
	53	53	53	53
S.F. Pot 34	10	29	09	18
	54	54	54	54
S.F. Pot 35	10	29	09	18
	55	55	55	55
S.F. Pot 36	10	29	09	18
	56	56	56	56
S.F. Pot 37	10	29	09	18
	57	57	57	57
S.F. Pot 38	10	29	09	18
	58	58	58	58
S.F. Pot 39	10	29	09	18
	59	59	59	59
S.F. Pot 40	10	29	09	18
	60	60	60	60

$\dfrac{a}{b} = \dfrac{\text{row}}{\text{column}}$	HIGH	LOW	ARM
I.C. Pot 1	17 01	18 01	19 01
I.C. Pot 2	17 02	18 02	19 02
I.C. Pot 3	17 03	18 03	19 03
I.C. Pot 4	17 04	18 04	19 04
I.C. Pot 5	17 05	18 05	19 05
I.C. Pot 6	17 06	18 06	19 06

a = row, b = column row/column = a/b	1 μf IN	1 μf OUT	1 μf IN	1 μf OUT	6 μf IN	6 μf OUT	10 μf IN	10 μf OUT	20 μf IN	20 μf OUT
Condensers	01 01	02 01	01 01 02 03	02 02 02 03	01 01 07 08	02 02 07 08	01 01 09 10	02 02 09 10	09 09 11 12	10 10 11 12
	03 01	04 01	03 03 02 03	04 04 02 03	03 03 07 08	04 04 07 08	03 03 09 10	04 04 09 10	11 11 11 12	12 12 11 12
	05 01	06 01	05 05 02 03	06 06 02 03	05 05 07 08	06 06 07 08	05 05 09 10	06 06 09 10	13 13 11 12	14 14 11 12
	07 01	08 01	07 07 02 03	08 08 02 03	07 07 07 08	08 08 07 08	07 07 09 10	08 08 09 10	15 15 11 12	16 16 11 12
	09 01	10 01	09 09 02 03	10 10 02 03	09 09 07 08	10 10 07 08	09 09 09 10	10 10 09 10		
	11 01	12 01	11 11 02 03	12 12 02 03	11 11 07 08	12 12 07 08	11 11 09 10	12 12 09 10		
	13 01	14 01	13 13 02 03	14 14 02 03	13 13 07 08	14 14 07 08	13 13 09 10	14 14 09 10		
	15 01	16 01	15 15 02 03	16 16 02 03	15 15 07 08	16 16 07 08	15 15 09 10	16 16 09 10		
			01 01 04 05	02 02 04 05			01 01 11 12	02 02 11 12		
			03 03 04 05	04 04 04 05			03 03 11 12	04 04 11 12		

a/b = row/column	1 μf		1 μf		9 μf		10 μf		20 μf	
	IN	OUT	IN	OUT	IN	OUT	IN	OUT	IN	OUT
Condensers			05 05 04 05	06 06 04 05			05 05 11 12	06 06 11 12		
			07 07 04 05	08 08 04 05			07 07 11 12	08 08 11 12		
			09 09 04 05	10 10 04 05						
			11 11 04 05	12 12 04 05						
			13 13 04 05	14 14 04 05						
			15 15 04 05	16 16 04 05						

$\frac{a = row}{b = column}$	10 Meg Ω	10 Meg Ω	4 Meg Ω	1 Meg Ω	1 Meg Ω	1 Meg Ω	0.25 Meg Ω	0.1 Meg Ω	0.1 Meg Ω
Resistors	03 03 13 14	04 04 13 14	05 05 13 14	06 06 13 14	07 07 13 14	08 08 13 14	09 09 13 14	10 10 13 14	11 11 13 14
	03 03 15 16	04 04 15 16	05 05 15 16	06 06 15 16	07 07 15 16	08 08 15 16	09 09 15 16	10 10 15 16	11 11 15 16
	03 03 17 18	04 04 17 18	05 05 17 18	06 06 17 18	07 07 17 18	08 08 17 18	09 09 17 18	10 10 17 18	11 11 17 18
	03 03 19 20	04 04 19 20	05 05 19 20	06 06 19 20	07 07 19 20	08 08 19 20	09 09 19 20	10 10 19 20	11 11 19 20

$$\frac{a}{b} = \frac{\text{row}}{\text{column}}$$

Hold Buss	25 01	25 02	25 03	25 04	25 05	25 06	25 07	25 08	25 09	25 10	25 11	25 12	25 13	25 14	25 15	25 16	25 17	25 18	25 19	25 20	
	25 61	25 62	25 63	25 64	25 65	25 66	25 67	25 68	25 69	25 70	25 71	25 72	25 73	25 74	25 75	25 76	25 77	25 78	25 79	25 80	17 80
Reset Buss	27 01	27 02	27 03	27 04	27 05	27 06	27 07	27 08	27 09	27 10	27 11	27 12	27 13	27 14	27 15	27 16	27 17	27 18	27 19	27 20	
	27 61	27 62	27 63	27 64	27 65	27 66	27 67	27 68	27 69	27 70	27 71	27 72	27 73	27 74	27 75	27 76	27 77	27 78	27 79	27 80	

Boost Resistor	29 01	29 02	29 03	29 04	29 05	29 06	29 07	29 08	29 09	29 10	29 11	29 12	29 13	29 14	29 15	29 16	29 17	29 18	29 19	29 20

$$\frac{a}{b} = \frac{\text{row}}{\text{column}}$$

+100 Volt Buss	28/21	28/22	28/23	28/24	28/25	28/26	28/27	28/28	28/29	28/30	28/31	28/32	28/33	28/34	28/35	28/36	28/37	28/38	28/39	28/40
	28/41	28/42	28/43	28/44	28/45	28/46	28/47	28/48	28/49	28/50	28/51	28/52	28/53	28/54	28/55	28/56	28/57	28/58	28/59	28/60
	28/61	28/62	28/63	28/64	28/65	28/66	28/67	28/68	28/69	28/70	28/71	28/72	28/73	28/74	28/75	28/76	28/77	28/78	28/79	28/80
	01/77	01/78	01/79	01/80																
−100 Volt Buss	30/21	30/22	30/23	30/24	30/25	30/26	30/27	30/28	30/29	30/30	30/31	30/32	30/33	30/34	30/35	30/36	30/37	30/38	30/39	30/40
	30/41	30/42	30/43	30/44	30/45	30/46	30/47	30/48	30/49	30/50	30/51	30/52	30/53	30/54	30/55	30/56	30/57	30/58	30/59	30/60
	32/61	32/62	32/63	32/64	32/65	32/66	32/67	32/68	32/69	32/70	32/71	32/72	32/73	32/74	32/75	32/76	32/77	32/78	32/79	32/80
	05/77	05/78	05/79	05/80																

$\frac{a}{b} = \frac{\text{row}}{\text{column}}$

Meter	-	+
	13 19	14 19

XT Pot	HIGH	LOW	ARM
	12 20	13 20	14 20

Ext. Term	1	2	3	4	5	6
	12 13	12 14	12 15	12 16	12 17	12 18

Read-Out Swtich	15 19	15 20	16 19	16 20	15 61	15 62	15 63	15 64

Diode	PLATE	CATHODE
	16 77	16 78
	16 79	16 80

Limiter	NI	+	-	OUT
	16 61	16 62	16 63	16 64
	16 65	16 66	16 67	16 68
	16 69	16 70	16 71	16 72
	16 73	16 74	16 75	16 76

	LM		Coil		RM	
	-	+			-	+
Esterline-Angus	14 65	15 65	14 66	15 66	14 67	15 67
	14 68	15 68	14 69	15 69	14 70	15 70
	14 71	15 71	14 72	15 72	14 73	15 73

$$\frac{a}{b} = \frac{\text{row}}{\text{column}}$$

GROUNDS

PANEL 1

12	17	18	19																
19	11	11	11																
34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34	34
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20

PANEL 2

21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
34	34	34	34																
21	22	23	24																

PANEL 3

21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
25	25	25	25	25	25	25	25												
41	42	43	44	45	46	47	48												

$$\frac{a}{b} = \frac{\text{row}}{\text{column}}$$

GROUNDS

02 61	02 62	02 63	02 64	02 65	02 66	02 67	02 68	02 69	02 70	02 71	02 72	02 73	02 74	02 75	02 76				
03 77	03 78	03 79	03 80																
04 61	04 62	04 63	04 64	04 65	04 66	04 67	04 68	04 69	04 70	04 71	04 72	04 73	04 74	04 75	04 76				
06 61	06 62	06 63	06 64	06 65	06 66	06 67	06 68	06 69	06 70	06 71	06 72	06 73	06 74	06 75	06 76				
07 77	07 78	07 79	07 80																
13 65	13 66	13 67	13 68	13 69	13 70	13 71	13 72	13 73											
17 70	18 70	19 70	20 70																
30 61	30 62	30 63	30 64	30 65	30 66	30 67	30 68	30 69	30 70	30 71	30 72	30 73	30 74	30 75	30 76	30 77	30 78	30 79	30 80
34 61	34 62	34 63	34 64	34 65	34 66	34 67	34 68	34 69	34 70	34 71	34 72	34 73	34 74	34 75	34 76	34 77	34 78	34 79	34 80

row = a / column = b	24 VOLTS	24 VOLTS	24 VOLTS	HOLD	OPERATE	RESET
24 Volt Relay Power Supply	17 07	17 79	19 79	17 08	18 08	19 08
	18 07	18 79	20 79	18 80	19 80	20 80
	19 07					

$\dfrac{\text{row}}{\text{column}}$ $\begin{matrix}a=\\b=\end{matrix}$	ARM	POSITION 1	POSITION 2	POSITION 3	POSITION 4	POSITION 5	POSITION 6	POSITION 7	POSITION 8	POSITION 9	POSITION 10	POSITION 11
Function Switch I	13 18	13 13	13 14	13 15	13 16	13 17						
	14 18	14 13	14 14	14 15	14 16	14 17						
	15 18	15 13	15 14	15 15	15 16	15 17						
	16 18	16 13	16 14	16 15	16 16	16 17						
Function Switch II	17 78	17 73	17 74	17 75	17 76	17 77						
	18 78	18 73	18 74	18 75	18 76	18 77						
	19 78	19 73	19 74	19 75	19 76	19 77						
	20 78	20 73	20 74	20 75	20 76	20 77						
Function Switch III	24 60	24 49	24 50	24 51	24 52	24 53	24 54	24 55	24 56	24 57	24 58	24 59
	25 60	25 49	25 50	25 51	25 52	25 53	25 54	25 55	25 56	25 57	25 58	25 59
	26 60	26 49	26 50	26 51	26 52	26 53	26 54	26 55	26 56	26 57	26 58	26 59

$\frac{a = \text{row}}{b = \text{column}}$	COIL	NO	A	NC	NO	A	NC	NO	A	NC
Relay	17 17 09 10	17 12	17 13	17 14	17 15	17 16	17 17	17 18	17 19	17 20
	18 18 09 10	18 12	18 13	18 14	18 15	18 16	18 17	18 18	18 19	18 20
	19 19 09 10	19 12	19 13	19 14	19 15	19 16	19 17	19 18	19 19	19 20
	17 17 71 72	17 61	17 62	17 63	17 64	17 65	17 66	17 67	17 68	17 69
	18 18 71 72	18 61	18 62	18 63	18 64	18 65	18 66	18 67	18 68	18 69
	19 19 71 72	19 61	19 62	19 63	19 64	19 65	19 66	19 67	19 68	19 69
	20 20 71 72	20 61	20 62	20 63	20 64	20 65	20 66	20 67	20 68	20 69

PANEL 4

| $\dfrac{a}{b} = \dfrac{row}{column}$ | | | | | | SPARE PLUGHOLES | | | | | | | | | | | | | |

07 61	07 62	07 63	07 64	07 65	07 66	07 67	07 68	07 69	07 70	07 71	07 72	07 73	07 74	07 75	07 76				
08 61	08 62	08 63	08 64	08 65	08 66	08 67	08 68	08 69	08 70	08 71	08 72	08 73	08 74	08 75	08 76	08 77	08 78	08 79	08 80
09 61	09 62	09 63	09 64	09 65	09 66	09 67	09 68	09 69	09 70	09 71	09 72	09 73	09 74	09 75	09 76	09 77	09 78	09 79	09 80
10 61	10 62	10 63	10 64	10 65	10 66	10 67	10 68	10 69	10 70	10 71	10 72	10 73	10 74	10 75	10 76	10 77	10 78	10 79	10 80
11 61	11 62	11 63	11 64	11 65	11 66	11 67	11 68	11 69	11 70	11 71	11 72	11 73	11 74	11 75	11 76	11 77	11 78	11 79	11 80
12 61	12 62	12 63	12 64	12 65	12 66	12 67	12 68	12 69	12 70	12 71	12 72	12 73	12 74	12 75	12 76	12 77	12 78	12 79	12 80
13 61	13 62	13 63	13 64	13 74	13 75	13 76	13 77	13 78	13 79	13 80									
14 61	14 62	14 63	14 64	14 74	14 75	14 76	14 77	14 78	14 79	14 80									
15 74	15 75	15 76	15 77	15 78	15 79	15 80													

When writing detailed instructions for wiring the plugboard, it is convenient to let the symbols in the circuit diagram represent the control wiring and to specify the input output wiring as follows:

1) Label each component symbol to indicate the column in which its inputs and outputs occur.

2) Label each input and each output to indicate the row in which it occurs.

This labeling specifies the location by row and column of both ends of all wires connecting inputs and outputs.

Instructions for wiring are written on modified plugboard diagrams. (See sample modified plugboard diagram.) In each space located at the coordinate of the plughole it represents are written the coordinates of the plughole to which it is connected. Labeling both input and output spaces creates a sort of double entry system in which the two plugholes a wire connects are indicated at both ends of the wire. A single entry system may be obtained by writing in only the input spaces or only the output spaces. When adjacent holes on the plugboard are to be connected by "bottle plugs", it is easier to indicate the connection by a line ($\int$) than to write out the specification as described.

The circuit diagram of problem 50.16 is marked to illustrate this method of labeling a circuit diagram to specify the input and output connections. All components in this circuit are performing standard operations. (see **Figure 77**).

Standard operations of the components are defined as follows:

1) D.C. Amplifiers 1 to 8 are used as integrators with 1 microfarad condensers in the feedback loop. Initial condition pots are connected to integrators 1 to 6.

2) D.C. Amplifiers 9 to 23 are used as summing amplifiers with 1 megohm resistors in the feedback loops.

3) Servos 1 to 16, Input tables 1 to 5, and the output table are connected as summing servos with +100 volts and -100 volts across the follow-up pots.

The connections for these operations are made with bottle plugs by connecting:

```
          Row 19 to Row 20 in Columns  1 to  6
          Row 22 to Row 23 in Columns  1 to 20 and 61 to 63
          Row 24 to Row 25 in Columns  1 to  8
          Row 26 to Row 27 in Columns  1 to  8
          Row 31 to Row 32 in Columns  1 to 24
          Row 33 to Row 34 in Columns  1 to 24
          Row 28 to Row 29 in Columns 61 to 80
          Row 31 to Row 32 in Columns 61 to 80
          Row 33 to Row 34 in Columns 61 to 80
          Row  1 to Row  2 in Columns 77 to 80
          Row  4 to Row  5 in Columns 77 to 80
  and Row 17 to Row 18 in Column  80
```

Most of the components in the circuit for any problem will be
performing standard operations. It is easier to specify the
departures from standard operation than to specify the control
wiring for all components in the circuit; therefore, all wiring
in the control section is assumed to be standard except changes
that are specified on the Wiring Changes sheet. See sample
Wiring Changes sheet following sample problem 50.16.

Actual wiring of the plugboard is done from the detailed wiring
instructions written on the modified plugboard diagram and the wiring
change sheet. A sort of running check on the wiring and the
instructions is set up by writing double entry instructions and
wiring the board systematically. If the board is wired column by
column from 1 to 80, then as the wiring progresses it is observed
that all wiring to lower numbered columns should have been completed.
This simple check should reveal single entries in the wiring
instructions and omission of wiring to higher numbered columns.
A similar check of wiring complete within a column is made by wiring
each column systematically row by row. A complete check of the
wiring is made by reversing the plugboard and using a buzzer to verify
the electrical connection between specified plugholes.

$$g = \int g \frac{dg}{d\eta} \, dt$$

$$\eta = \int g \, dt$$

$$\frac{dg}{d\eta} = -\int \frac{\rho\mu}{\rho_\infty \, \rho_\infty} \cdot \eta \, dt$$

$$\frac{dj}{dt} = Pr \int \left[\frac{dg}{d\eta} \cdot \frac{dj}{dt} - g^2 \right] dt$$

$$j = \int \frac{dj}{dt} \, dt$$

Fig. 77—Circuit diagram

AMPLIFIER OUTPUT

TO ITR S.J. OTO OTP

RESISTORS

EXT TERMINALS

FUNCTION SWITCH I

METER STAT

READ SW

I.C. POT   24 VOLT   H   GND   SELECTORS

| | | | | OP | | GNU | NO | ARM | NC | NO | ARM | NC | NO | ARM | NC |

R   GND

INITIAL CONDITION INPUT

D.C. AMPLIFIER SUMMING JUNCTION

D.C. AMPLIFIER GRID

FEED BACK CONDENSER   FEEDBACK RESISTOR

HOLD RELAY

HOLD BUSS

RESET RELAY

RESET BUSS

D.C. AMPLIFIER OUTPUT

BOOST RESISTOR

SERVO SUMMING JUNCTION   INPUT TABLE S.J.

FOLLOW-UP POT ARM   ARM

FEEDBACK RESISTOR   F.R.

SECONDARY INPUT   S.I.

GROUND BUSS

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
1			03,34				A POT ARM			03,33							03,35	03,23	1.T SW ARMS		1
2							B POT ARM			03,31											2
3			01,38		12,35		D.C. AMPLIFIER INPUTS			02,30			01,30	01,23	01,37						3
4																					4
5																					5
6																					6
7				11,34																	7
8	09,31	10,21		09,33						12,21		12,23				13,35					8
9					15,31		SF POT ARM			08,21			08,24								9
10	08,22						SF POT HI														10
11		24,59					D.C. AMPLIFIER OUTPUT			24,58				07,24							11
12	08,30		08,32												03,25						12
13						14,59									08,36						13
14							SERVO INPUTS										18,37	18,23	1.T INPUTS		14
15																	09,25	18,38			15
16																					16
17																					17
18					14,38		S.F. POT ARM										14,37	15,38			18
19		20,38		20,37			D.C. AMPLIFIER OUTPUT									20,23					19
20			19,36				A POT HI										19,24	19,22	SW HI		20
21							GROUND BUSS														21
22			23,35				A POT LO			23,21								1.T SW LO			22
23	23,30		24,30				D.C. AMPLIFIER OUTPUT								22,23						23
24							B POT HI			23,23								1.T POT HI			24
25							GROUND BUSS														25
26							B POT LO			27,32								1T POT LO			26
27							D.C. AMPLIFIER OUTPUT			26,30											27
28							+100 V BUSS														28
29							SF POT HI														29
30							−100V BUSS														30

	ITG	TFP	OTG	QTP															
31	F POT ARM				COIL		NO	ARM	DIFFERENTIAL RELAYS	NO	ARM	NO							
32	F.B RESISTOR				COIL		NO	ARM	NO		COIL		NO	ARM	NO				
33	SEC INPUT				COIL		NO	ARM	NO		COIL		NO	ARM	NO				
34	GROUND																		

Columns: 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

Row 1: C POT ARM I.T. SW ARM
Row 2: L_1 | R_1 | L_2 | R_2 | L_9 | R_9 | L_{10} | R_{10} | I.T. 5 SW ARM I | I.T. POT ARM
Row 3: D.C. AMPLIFIER INPUTS

Row 9: S.F POT ARM
Row 10: S.F POT HI

Row 11: D.C AMPLIFIER OUTPUT

Row 14: F POT ARM ITD ITF OTD OTP
  2,26 4,60

Row 17: X | Y | SIN | COS | X | Y | SIN | COS | X | Y | SIN | COS | X | Y | SIN | COS | 2,
Row 18: S.F. POT ARM
Row 19: D.C. AMPLIFIER OUTPUT
Row 20: C POT HI IT 5 O.T.

Row 21: GROUND BUSS
Row 22: C POT LO SW POT
Row 23: D.C AMPLIFIER OUTPUT
Row 24: L_1 | R | L SERVO SW HI R | L | R | FUNCTION SWITCH III | 11,30 11,22 4,60
Row 25: GROUND | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | A

Row 26: L | R | L SERVO SW LO R | L | R
Row 27: D.C. AMPLIFIER OUTPUT
Row 28: +100 VOLT BUSS
Row 29: S.F. POT HI
Row 30: -100 VOLT BUSS

Columns: 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

Column headers: 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

Row	Center label	Right-side notes
1	A POT CT	+100V BUSS
2	GROUND BUSS	F HI — ITP ITSP STD OTP
3	B POT CT	GND
4	GROUND BUSS	F LO
5	C POT CT	−100V BUSS
6	GROUND BUSS	CENTER TAPS
7		GND
8		
9		
10		
11		
12		
13	GROUND BUSS	
14	ESTERLINE ANGUS	
15	READ OUT SWITCH — LM − COIL + RM — LM — COIL — RM — LM — COIL + RM	
16	LIMITERS — IN − + OUT IN − + OUT IN − + OUT	
17	SELECTORS — GND	FUNCTION SW II — 24V K.B.
18	NO ARM NC NO ARM NC NO ARM NO GND — 1 2 3 4 5 ARM	24V HOLD
19	GND	24V OF
20	GND	24V RESET
21	D.C. AMPLIFIER SUMMING JUNCTION	
22	D.C. AMPLIFIER GRID	
23	FEEDBACK RESISTOR	
24	HOLD RELAY	
25	HOLD BUSS	
26	RESET RELAY	
27	RESET BUSS	
28	+100 VOLT BUSS	
29	A POT HI	1C F HI
30	GROUND BUSS	
31	A POT LO	1C F LO
32	−100 VOLT BUSS	
33	F POT CT	1C F CT
34	GROUND BUSS	

WIRING CHANGES   PROBLEM_____   PAGE____

CHANGE NUMBER	REMARKS	REMOVE		CONNECT	
		FROM	TO	FROM	TO
CHANGE NUMBER	REMARKS	REMOVE FROM	TO	CONNECT FROM	TO

V. OPERATING CONTROLS

The operating controls can be understood best by studying Figures 78 through 80 which identify the major operating controls and list their functions and modes of operation.

The first step in the use of the REAC is to test the machine with the checking boards as discussed in Section VI H, after which the problem board is inserted. The potentiometers are then set by flipping the "pot set" switch under the digital readout and setting each potentiometer by reading the output selected by the readout selector switches under the output table. Following this the "pot set" switch is returned to the "off" position, turning off the red warning light.

The next step is the test of the input tables by plotting the functions on the output table. ·Next comes the static test in which reasonable values are placed on all integrators and potentiometers and the output of every amplifier, constant factor potentiometer, servo-potentiometer, input table, etc., is checked against precomputed values. If all is well, insert the graph paper and start plotting.

The left-hand switch under the digital readout is for reducing the magnitude of the readout by 100 to permit the reading of voltages over 100 volts. When the switch is in this abnormal position an adjacent green warning light glows. The switch is thrown up when $+100 < e_i \leq 200$ V and down when $-200 \leq e_i < -100$ V.

The main controls are the "Reset", "Operate", and "Hold" push buttons; adjacent lights indicate which control is in effect.
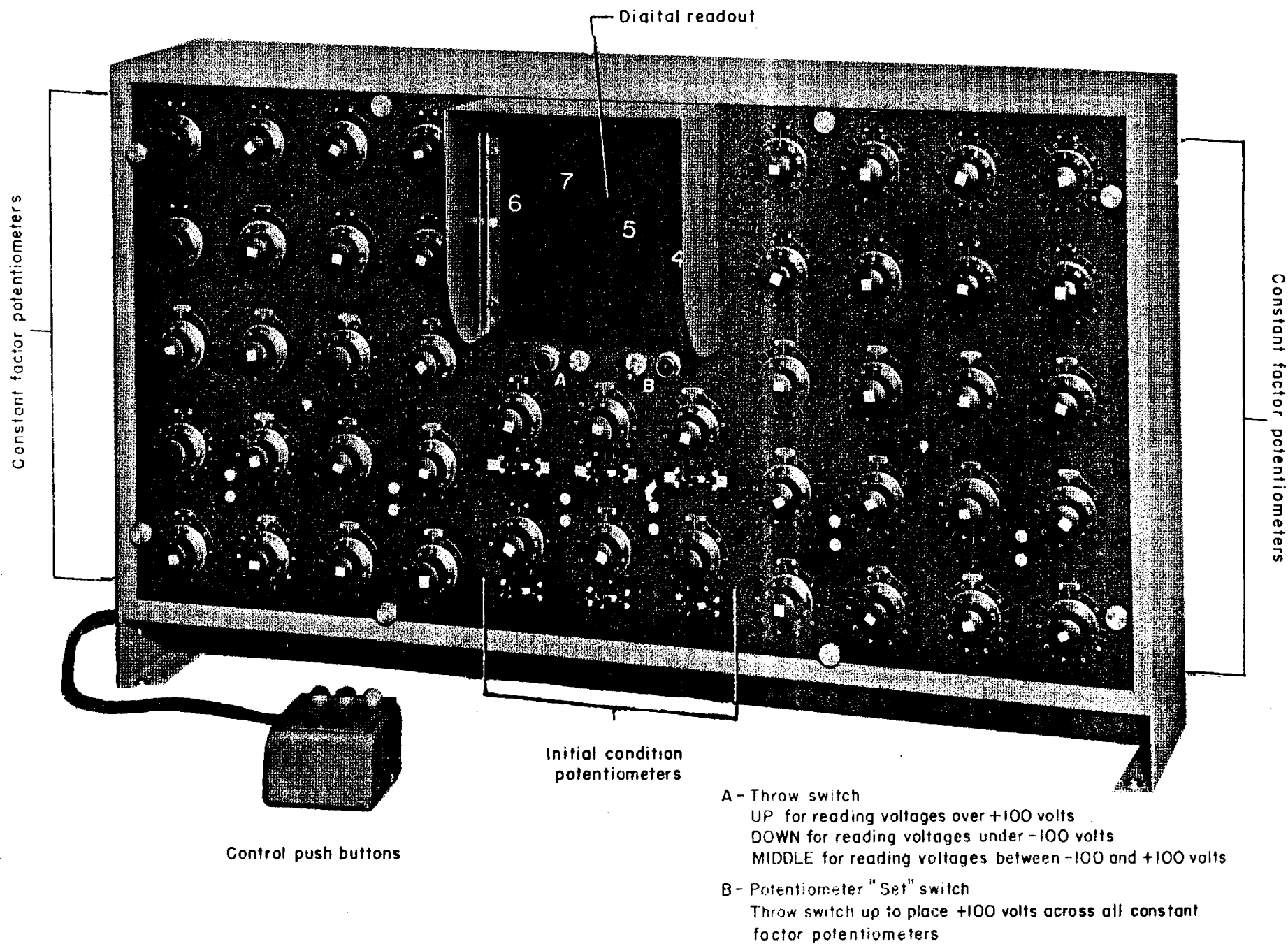
Constant factor potentiometers

Constant factor potentiometers

7
6
5
4

Initial condition
potentiometers

Control push buttons

A – Throw switch
UP for reading voltages over +100 volts
DOWN for reading voltages under –100 volts
MIDDLE for reading voltages between –100 and +100 volts

B – Potentiometer "Set" switch
Throw switch up to place +100 volts across all constant
factor potentiometers

Fig. 78 — Potentiometer and digital readout panel

Output table

Function switches

Readout switch

Selector switch

Fig. 79 — Switching panel

Amplifier manual
balance controls

Servo response controls

Gain controls

Damping controls

Critical damping switches

Gain controls

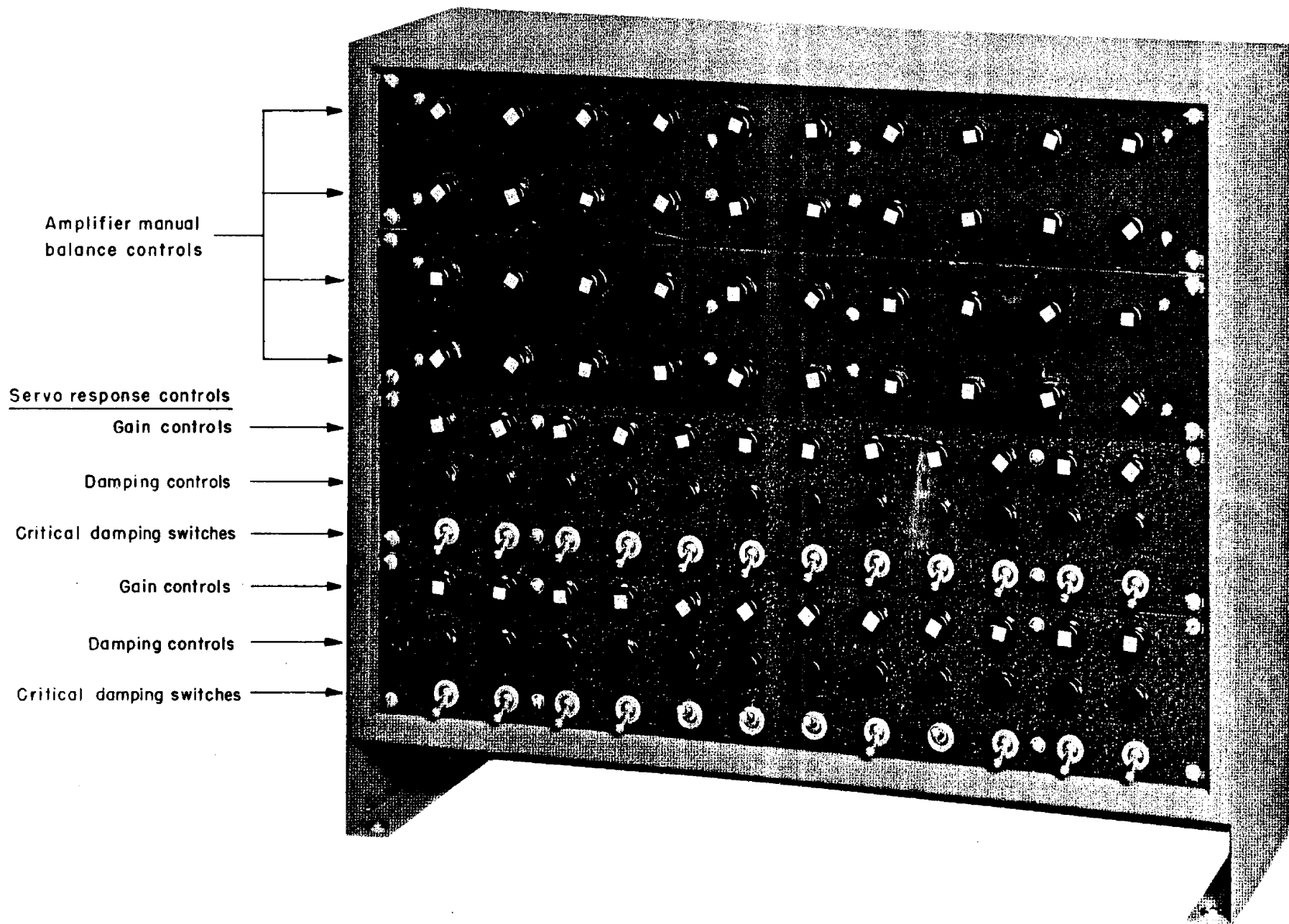Damping controls

Critical damping switches

Fig. 80 — Manual balance and servo operating control panel

VI. TESTING AND ADJUSTING MACHINE OPERATION

The testing and adjusting of the computer usually is done by the maintenance staff, but the operator should be acquainted with the following techniques to speed up maintenance and to assure himself of proper machine operation.

The ± 100 volt power supply is the most accurate element in the computer, the magnitude of the two voltages being within ± 0.001 volt. The high precision, 40-turn helipot is the next most accurate component with a guaranteed linearity of ± 0.03 per cent and the actual settings known to $\pm 0,01$ per cent at 80 points. After a check that these two components are operating properly, they are instrumental in checking other components. The latest calibration tables for the 40-turn potentiometer are given in Tables VIII and IX.

TABLE VIII. CALIBRATION OF 40 TURN POT FOR EQUAL RATIO INCREMENTS

Ratio	Dial	Ratio	Dial
.0125	.496	.5125	20.506
.0250	.994	.5250	21.001
.0375	1.499	.5375	21.502
.0500	1.994	.5500	22.003
.0625	2.492	.5625	22.499
.0750	2.989	.5750	22.996
.0875	3.488	.5875	23.503
.1000	3.990	.6000	24.000
.1125	4.488	.6125	24.497
.1250	4.987	.6250	24.994
.1375	5.484	.6375	25.495
.1500	5.979	.6500	25.992
.1625	6.484	.6625	26.492
.1750	6.985	.6750	26.991
.1875	7.487	.6875	27.492
.2000	7.987	.7000	27.993
.2125	8.484	.7125	28.492
.2250	8.986	.7250	28.989
.2375	9.490	.7375	29.486
.2500	9.986	.7500	29.986
.2625	10.491	.7625	30.486
.2750	10.993	.7750	30.986
.2875	11.491	.7875	31.490
.3000	11.991	.8000	31.984
.3125	12.490	.8125	32.485
.3250	12.994	.8250	32.986
.3375	13.496	.8375	33.483
.3500	14.000	.8500	33.986
.3625	14.500	.8625	34.484
.3750	14.997	.8750	34.982
.3875	15.499	.8875	35.486
.4000	16.000	.9000	35.985
.4125	16.501	.9125	36.482
.4250	17.003	.9250	36.983
.4375	17.502	.9375	37.484
.4500	18.000	.9500	37.989
.4625	18.504	.9625	38.490
.4750	18.997	.9750	38.988
.4875	19.500	.9875	39.501
.5000	19.999	1.0000	40.001

TABLE IX. CALIBRATION OF 40 TURN POT FOR EQUAL DIAL INCREMENTS

Dial	Ratio	Error x 10^{-5}	Dial	Ratio	Error x 10^{-5}
0.5	.01261	11	20.5	.51236	-14
1.0	.02515	15	21.0	.52498	- 2
1.5	.03765	15	21.5	.53746	- 4
2.0	.05016	16	22.0	.54993	- 7
2.5	.06269	19	22.5	.56252	2
3.0	.07528	28	23.0	.57509	9
3.5	.08780	30	23.5	.58742	- 8
4.0	.10026	26	24.0	.60000	0
4.5	.11281	31	24.5	.61258	8
5.0	.12532	32	25.0	.62515	15
5.5	.13789	39	25.5	.63763	13
6.0	.15053	53	26.0	.65021	21
6.5	.16289	39	26.5	.66269	19
7.0	.17538	38	27.0	.67522	22
7.5	.18782	32	27.5	.68769	19
8.0	.20033	33	28.0	.70017	17
8.5	.21289	39	28.5	.71270	20
9.0	.22533	33	29.0	.72528	28
9.5	.27775	25	29.5	.73776	26
10.0	.25035	35	30.0	.75038	38
10.5	.26273	23	30.5	.76286	36
11.0	.27517	17	31.0	.77534	34
11.5	.28772	22	31.5	.78777	27
12.0	.30022	22	32.0	.80039	39
12.5	.31275	25	32.5	.81287	37
13.0	.32516	16	33.0	.82535	35
13.5	.33761	11	33.5	.83793	43
14.0	.35000	0	34.0	.85036	36
14.5	.36249	- 1	34.5	.86289	39
15.0	.37507	7	35.0	.87546	46
15.5	.38752	2	35.5	.88784	34
16.0	.40001	1	36.0	.90037	37
16.5	.41248	- 2	36.5	.91295	45
17.0	.42493	- 7	37.0	.92543	43
17.5	.43744	- 6	37.5	.93785	35
18.0	.44996	- 1	38.0	.95028	28
18.5	.46239	-11	38.5	.96276	26
19.0	.47507	7	39.0	.97529	29
19.5	.48750	0	39.5	.98747	- 3
20.0	.50002	2	40.0	.99997	- 3

A. Power Supply

Although the ±100 volt power supply is the most accurate components of the computer, it should be checked occasionally to assure proper operation. The magnitude of the ±100 supply is not important, but the two voltages should be held within at least ±0.01 volt of each other. The circuit of Figure 81 not only checks the power supply, but also spot checks the 40-turn potentiometer. In both settings of the switch the helipot is adjusted to give zero current through the milliammeter. Letting

$$\left(\frac{V-}{V+}\right)_A = \frac{1 - k_A}{k_A} \quad \text{in position A}$$

$$\left(\frac{V-}{V+}\right)_B = \frac{k_B}{1 - k_B} \quad \text{in position B}$$

it can be shown that

$$\frac{V-}{V+} = \frac{1}{2}\left[\left(\frac{V-}{V+}\right)_B + \left(\frac{V-}{V+}\right)_A\right]$$

and

$$\text{Error in potentiometer setting} = \frac{1}{2}\left[\left(\frac{V-}{V+}\right)_B - \left(\frac{V-}{V+}\right)_A\right]$$
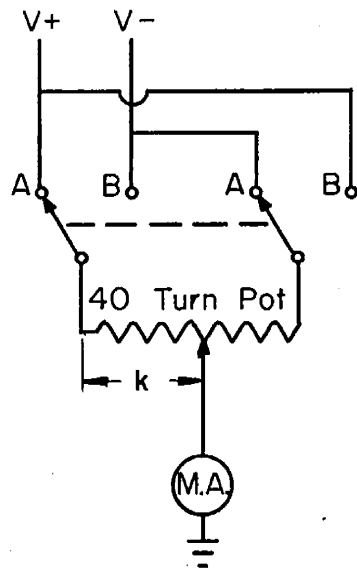
Fig. 81 ——Circuit for checking power supply
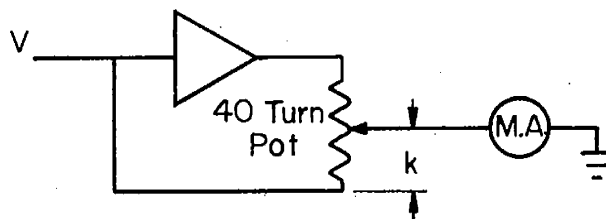
Gain = $\frac{1}{k} - 1$

(a) Computing amplifier
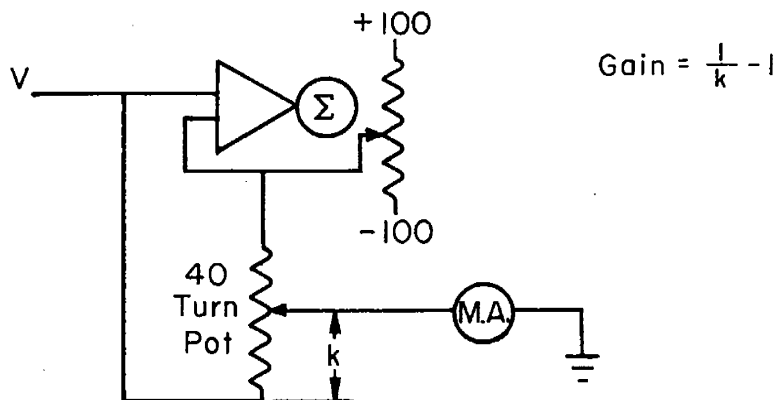
(b) Servo amplifier

Fig. 82 ——Circuit for testing amplifier gains

Resistor under test

R_T

Standard
Resistor

R_S

V

G

k

M.A

$$\frac{R_T}{R_S} = \frac{(1-k)}{k} = \frac{1}{k} - 1$$
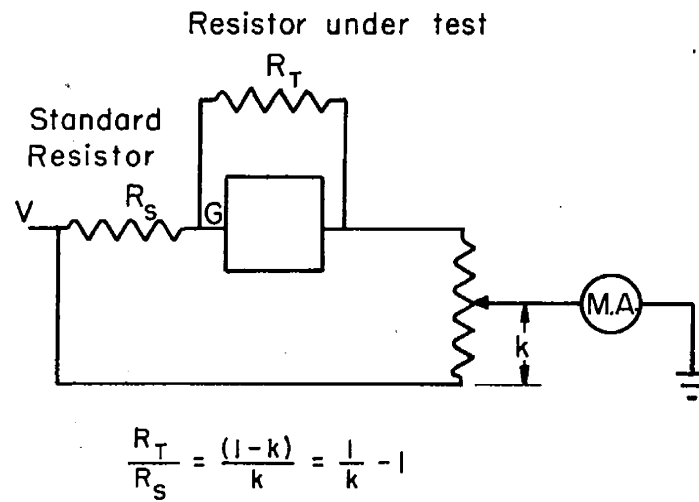
Fig. 83—Circuit for finding relative resistance

Condenser under test

C_s

Standard
Condenser

C_T

-100 V

I.C. = 0

e_i

G

$-\frac{C_T}{C_S} e_i$

$e_0 = \left(\frac{C_T}{C_S} - K\right) e_i$

K

10

$-10\ e_0$

Operate until $e_i = \dfrac{100}{K}$ volts with $K \approx \dfrac{C_T}{C_S}$
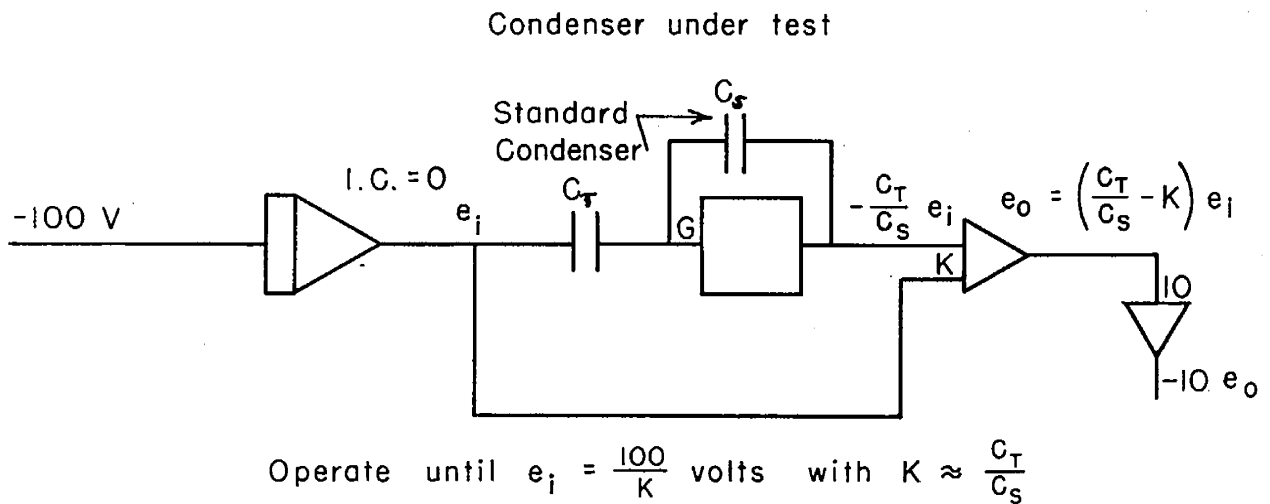
Then

$$\frac{C_T}{C_S} = K \left(1 + \frac{e_0}{100}\right)$$

Fig. 84 —— Circuit for finding relative capacitance

B.   Computing and Servo Amplifier Gains

The gains of the amplifiers can be checked by a circuit similar to that for testing the power supplies, as illustrated by Figure 82. When the precision potentiometer is set to make the meter current zero, the gain of the amplifier is given by

$$G = \frac{1 - k}{k}$$

A trimming resistor in series with whichever resistor is too small will correct the gain of an amplifier. Since these trimming resistors will be only a thousand ohms or so, inexpensive ten per cent resistors may be used.

The circuit of Figure 85 is useful for finding the per cent difference between two resistors of nearly the same resistance. This circuit requires that the magnitudes of the plus and minus 100 volt supplies be equal.

C.   Resistors and Condensers

The magnitude of the resistors and condensers are not important, but their values relative to each other should be kept to within 0.01 per cent if possible.

The relative value of a resistor can be found by using the circuit of Figure 83. If k_0 is the value of k for a test resistance of R_0 volts, where $R_0 \approx R_T$

$$R_T - R_0 \approx - \frac{(k - k_0)}{k_0^2}$$

Figure 84 gives the circuit used for finding the relative value of a condenser. It is obvious that the summing amplifier of Figure 83 is performing the same job as the potentiometer of Figure 84, i.e., comparing voltages of opposite sign. Hence, these portions of Figures 83 and 84 can be interchanged.
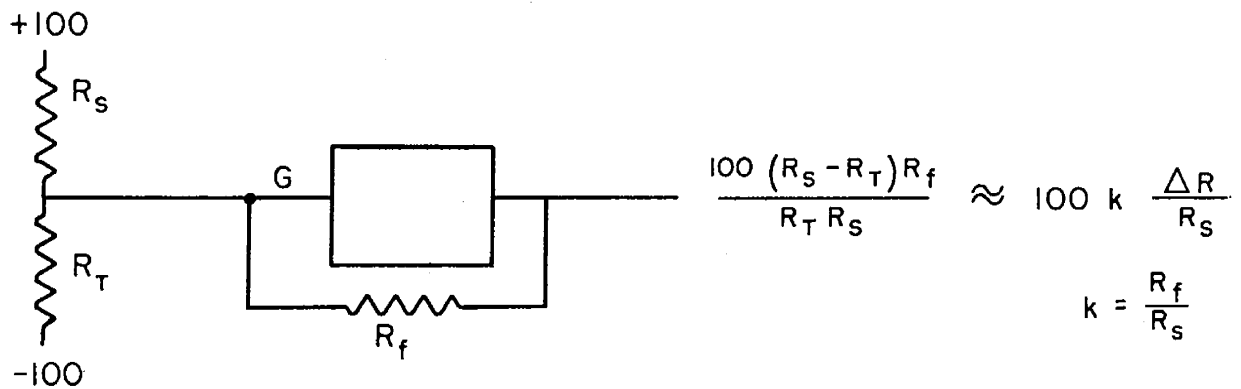
$$\frac{100\,(R_S - R_T)R_f}{R_T\,R_S} \approx 100\,k\;\frac{\triangle R}{R_S}$$

$$k = \frac{R_f}{R_S}$$

Fig. 85 —— Circuit for finding percent difference between two nearly equal resistors

D. Servo-Multiplier Potentiometer Linearity

The linearity of the servo-multiplier potentiometers is not particularly important - the relative error between the follow-up potentiometer and the coupled multiplying potentiometers is what really counts in accuracy of multiplication. The circuit of Figure 86 is convenient for plotting the relative potentiometer error.
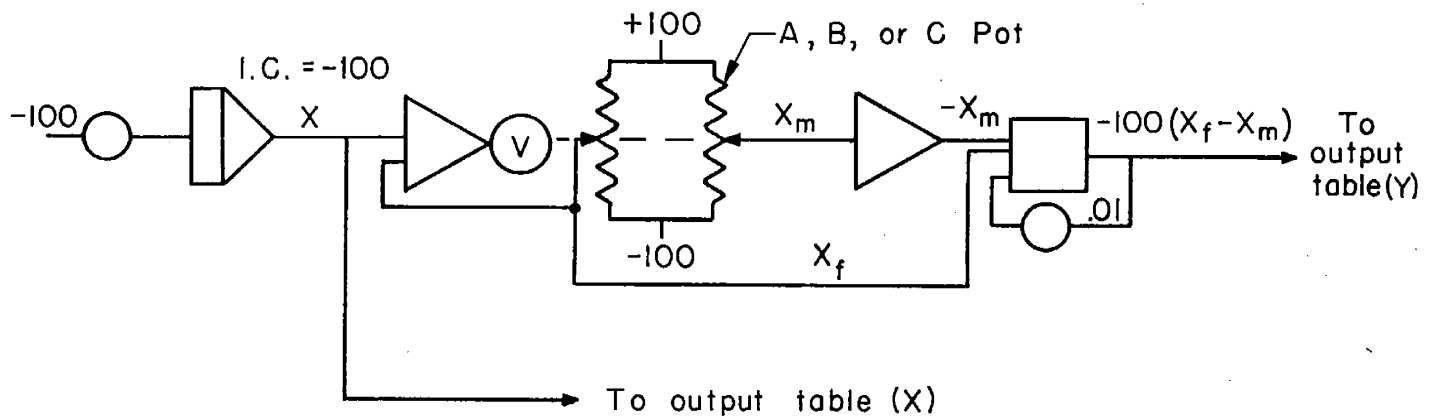
Fig. 86——Circuit for plotting potentiometer relative error

E. Input Table Linearity

A graph with a straight wire through the points
$f(x)$ = 100 V at x = +100 V and $f(x)$ = -100 V at x = -100 V
(plotting $f(x)$ = x) is useful for checking the linearity of an
input table. The slide wire is then tested relative to the input
table servo follow-up potentiometer by the same circuit used for
testing the multiplying potentiometers.

F. Output Table Linearity

The linearity of the output table can be tested by the
same voltage into the x and y inputs and varying the voltage
between plus and minus 100 volts. The resulting plot can be
checked with a straight edge. The smoothness of the table operation
shows up when $100\ e^{-at}\cos\omega t$ is fed into the x input and $100\ e^{-at}\sin\omega t$
into the y input. A similar test uses Lissajous figures, placing
$100\ \cos\omega t$ in one input and $100\ \cos k\omega t$ in the other.

G. Servo Response

The servo controls are mounted on the panel at the right
end on the console. The response of the servos will need adjusting
whenever the voltage across the follow-up potentiometer is changed.
The usual procedure is to increase the gain until the servo starts
to chatter and vary the damping until the servo has only one or
two overswings when responding to a step function.

H. Amplifier Drift

The total offset and leakage current should be such that
drift of an amplifier is less than 0.05 volt per minute with the
input grounded, a condenser feedback loop, and a gain of 20. The
zero-set controls are above the servo response controls, but should
be needed only when an amplifier is in manual balance operation.

The noise level of the amplifiers in the 10 - 100,000
cycle per second band should be less than one millivolt.

APPENDIX I. BELL LABORATORY TECHNIQUES

The following techniques have been lifted in a modified form from memoranda by Emery Lakatos of Bell Laboratory.

CALCULATION OF FOURIER SERIES COEFFICIENTS

The solution of the differential equation

$$\frac{d^2u}{d\theta^2} + n^2u = f(\theta)$$

$$u(0) = u'(0) = 0$$

evaluated at $\theta = 2\pi$ gives

$$u(2\pi) = -\frac{1}{n}\int_0^{2\pi} f(\lambda)\,\sin\,(n\lambda)\,d\lambda = -\frac{\pi}{n}\,a_n$$

$$u'(2\pi) = \frac{1}{n}\int_0^{2\pi} f(\lambda)\,\cos\,(n\lambda)\,d\lambda = \frac{\pi}{n}\,b_n\ ,$$

where a_n and b_n are the coefficients in the Fourier Series

$$f(\theta) = \frac{a_0}{2} + \sum_1^\infty a_n\,\cos\,n\theta + \sum_1^\infty b_n\,\sin\,n\theta\ .$$

Since only three amplifiers are required per set of two coefficients, several sets can be computed in one run.

In most cases $f(\theta)$ will be graphical. Let k be the number of inches per unit of $f(\theta)$ on the vertical axis of 10" x 15" graph paper. Place 100 volts across the input table slide wire, as shown in Figure 87. If the $f(\theta)$ zero axis is c inches from the bottom of the graph, the voltage on the slide wire $e(\theta)$ will be

$$e(\theta) = 10(c + kf(\theta))\ \text{volts.}$$

Using a solution time T of 62.8 seconds for $1 \le n \le 10$,

$$\frac{d\theta}{d\tau} = k = \frac{2\pi}{62.8} = 0.1$$

Fig. 87—Circuit for computing fourier coefficients

and the potentiometers are set at 0.1n as shown in Figure 87. For $10 < n \le 20$ use a solution time T of 125.6 seconds and set the potentiometers at 0.05n.

If one period of the graph extends d inches ($d_{max} = 15"$) and n is less than 11, the integrator is adjusted to take 62.8 seconds to reach $-100 \frac{d}{15}$ volts (the graph paper is adjusted to give $\theta = 0$ when $\tau = 0$). Hence, the setting of potentiometer 3 is $.159\left(\frac{d}{15}\right)$.

Calling V_1 the output in volts of integrator 1 and V_2 the output of integrator 2 at $\theta = 2\pi$,

$$na_n = \frac{0.1 V_2}{k\pi}$$

$$nb_n = \frac{0.1 V_1}{k\pi}$$

To compute $\frac{a_0}{2}$, connect the output of the input table directly to an integrator. Calling the integrator voltage at $\theta = 2\pi$, V_3,

$$\frac{a_0}{2} = \frac{1}{k} \left[\frac{V_3}{628G} - c \right]$$

where G is the gain of the integrator.

TRANSFER FUNCTIONS

Many physical systems obey equations of the type

$$\frac{V(t)}{E(t)} = Y(p) = \frac{a_0 + a_1 p + a_2 p^2 + \ldots + a_m p^m}{b_0 + b_1 p + b_2 p^2 + \ldots + a_n p^n} , \quad n \ge m$$

where E(t) is the driving force, V(t) is the response, p is the differential operator $\frac{d}{dt}$, the Y(p) is the transfer function of the system.

Changing the above equation to implicit form, collecting terms, and dividing by p^n we obtain
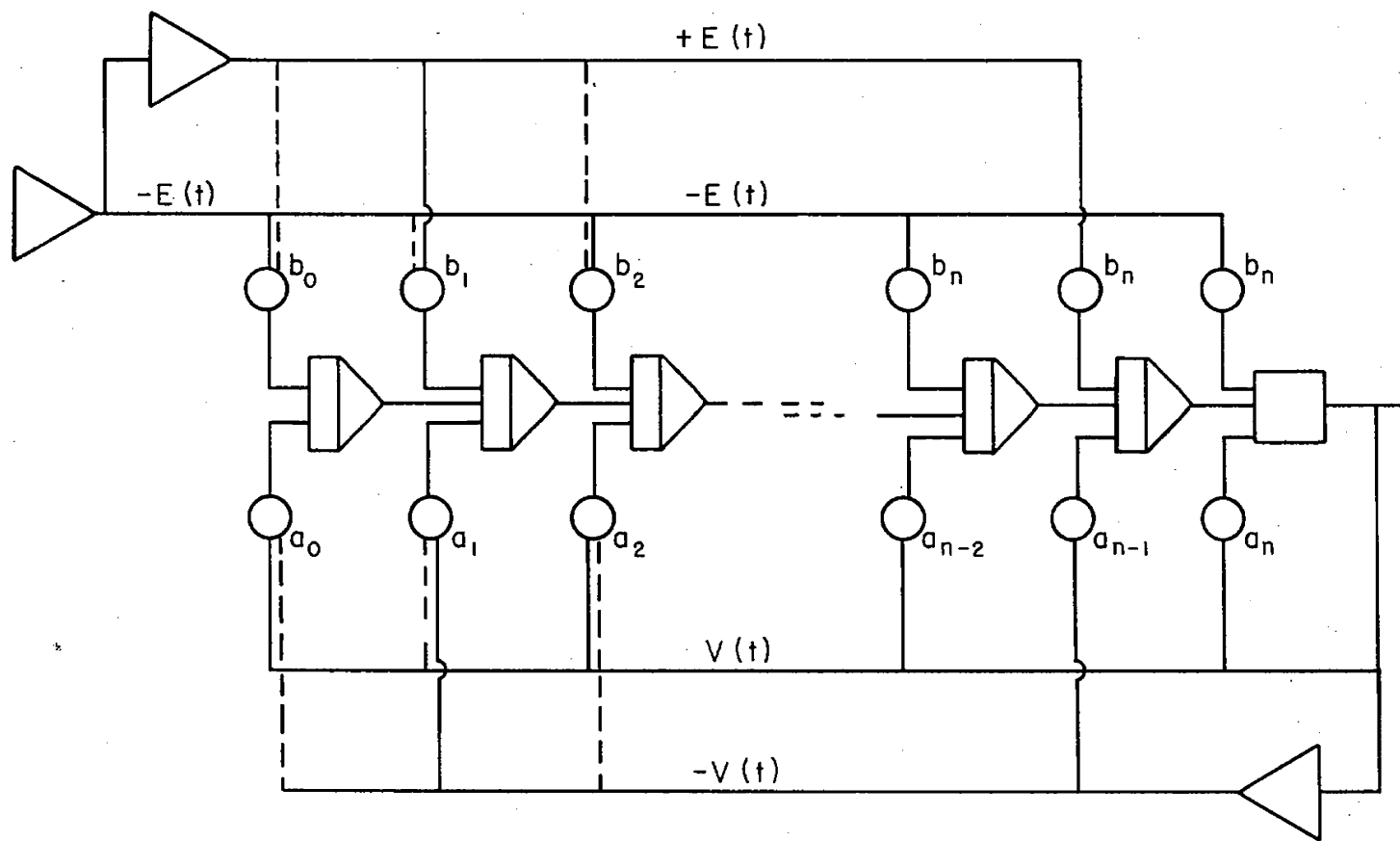
$$V(t) = Y(p) \cdot E(t)$$

$$Y(p) = \frac{a_0 + a_1 p + a_2 p^2 + \cdots + a_m p^m}{b_0 + b_1 p + b_2 p^2 + \cdots + b_n p^n}$$

$$n \geq m$$

Dotted connections for n odd

Solid connections for n even

Fig. 88—Transfer function circuit

$$(b_nV - a_nE) + \frac{1}{p}(b_{n-1}V - a_{n-1}E) + \frac{1}{p^2}(b_{n-2}V - a_{n-2}E) + \ldots + \frac{1}{p^n}(b_0V - a_0E) = 0$$

This equation leads to the circuit of Figure 88. The principal advantage of this scheme is its economical use of amplifiers.

ROOTS OF REAL POLYNOMIALS *

Assume for the moment that the polynomial

$$f(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$$

has only conjugate complex roots, the real roots having been found previously and factored out.

If this polynomial is divided by the quadratic

$$x^2 + px + q$$

the remainder is of the form

$$R = g_n(p,q)x + h_n(p,q),$$

where g_n and h_n are polynomials in p and q. Hence, the quadratic factors may be found by solution of the simultaneous, implicit equations

$$g_n(p,q) = 0$$

$$h_n(p,q) = 0.$$

A study of the division process shows that $g_n(p,q)$ is generated by the following iterative process:

---

*The circuit of this section is essentially the same as Lakatos', but the analysis is different.

$$g_1 = 1$$

$$g_2 = a_{n-1} - pg_1$$

$$g_3 = a_{n-2} - pg_2 - qg_1$$

$$g_4 = a_{n-3} - pg_3 - qg_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$g_j = a_{n-j+1} - pg_{j-1} - qg_{j-2}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$g_n = a_1 - pg_{n-1} - qg_{n-2} \; .$$

The function $h_n(p,q)$ is given by

$$h_n(p,q) = a_0 - qg_{n-1} \; .$$

Figure 89, with S_1, S_2 and S_3 open and S_4 closed, mechanizes the above equations.

Since the roots are all conjugate complex, q is always positive. However, p may be of either sign. As drawn Figure 89 applies for positive values of p; negative values may be found by placing a negative value of V at position A of Figure 89.

A slight modification of the above circuit permits the computation of real roots. Dividing $f(x)$ by $x + p$ leaves a remainder term

$$\overline{h}_n = a_0 - pg_n$$

with $q \equiv 0$. Solving the implicit equation

$$\frac{a_0}{p} - g_n = 0$$

by closing S_1, S_2, and S_3 and opening S_4 gives the positive values
of p. Negative values of p require changing the sign of V. After
all real roots have been improved and removed, reset the A
potentiometers, reverse the switches and shift the plug-in point
of the P and Q leads to the left by as many amplifiers as there
were real roots.

It is necessary for scale factor determination to estimate
q_m, the maximum value of q, and p_m, the maximum value of p. The
machine variables will be $Q = \dfrac{q}{q_m}$ and $P = \dfrac{p}{p_m}$. Since the roots are
conjugate complex, we can safely let

$$p_m = 2 \sqrt{q_m} \, .$$

Consequently, the scale factor potentiometer settings are given by

$$A_j = \frac{a_j}{4} \left(\frac{2}{\sqrt{q_m}} \right)^{n-j}$$

$$j = 0, 1, 2, 3, 4, \ldots, n-1$$

and the amplifier gains are as shown in Figure 89. The voltage V
is set at the maximum value possible without overloading any
amplifier.

A good estimate of q_m can be found by Cauchy's test.
Take a small integer c and calculate the sequence

$$f_0 = 1$$

$$f_1 = c - |a_{n-1}|$$

$$f_2 = f_1 c - |a_{n-2}|$$

$$f_3 = f_2 c - |a_{n-3}|$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$f_j = f_{j-1} c - |a_{n-j}|$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$f_n = f_{n-1} c - |a_0| \; .$$

The smallest value of c that makes all the f's positive is an upper bound of $\sqrt{q_m}$. This test seems to give estimates that are three times too large. It is generally safe, therefore, to arbitrarily decrease the estimate by a factor of two.

$q_n (p,q)$

P plug-in point

Q plug-in point

$h_n (p,q)$
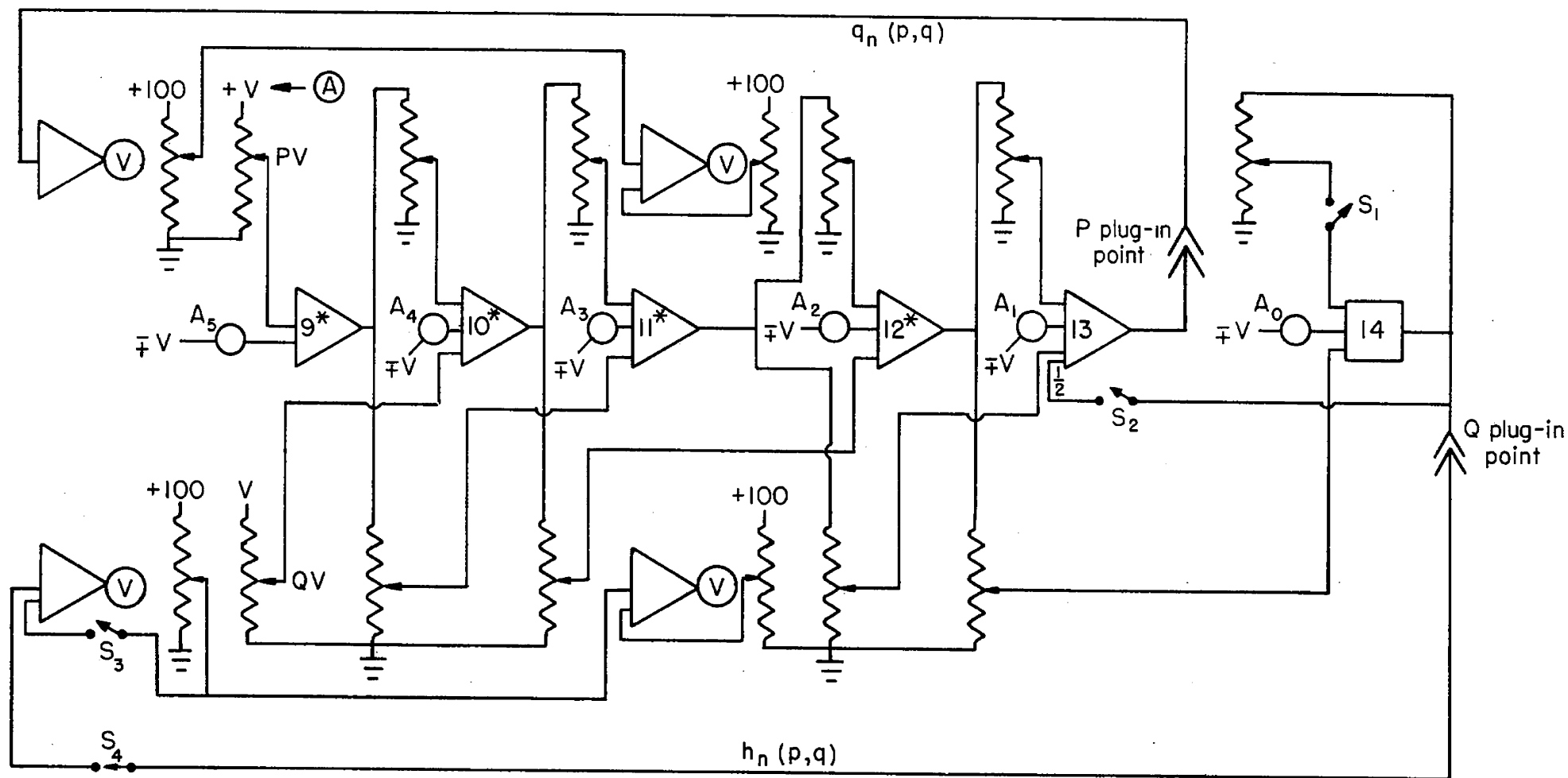
*Input gains of amplifiers 9 through 12 are set at two by replacing feedback resistors with 2 M resistors
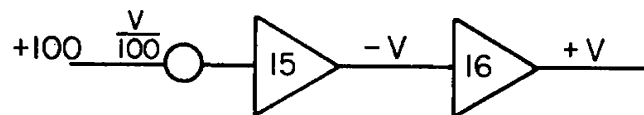
Switches as shown for complex roots
Switches reversed for real roots

Sign of V at point (A) as shown for P > 0
Sign of V reversed for P < 0

Fig. 89—Circuit for computing roots of sixth degree polynomial

APPENDIX II. TECHNIQUES USED IN RECENT PROBLEMS

A recent statistical problem was run with the REAC in self-cycling operation. A dearth of amplifiers required a non-conventional approach for the self-cycling and punching of results on IBM cards. The circuitry for this operation and two others involved in the problem are illustrated in Figure 90. One circuit, yielding a sine wave of constant amplitude, was a modification of a circuit suggested by Dr. Robert Bennett of Hughes Aircraft Company.

If it is essential that the positive and negative slopes of the sawtooth be of the same magnitudes, two poles of the relay must be used, one going to the integrator and the other to the high gain amplifier. Otherwise the high gain amplifier grid voltage (the order of 10 volts) will be coupled to the integrator when the relay is in the NO position.

As one portion of the statistical problem it was desired to find the maximum value of a random noise. Since only one amplifier was available, the circuit of Figure 63 could not be used. The circuit of Figure 91 was used and worked quite well.
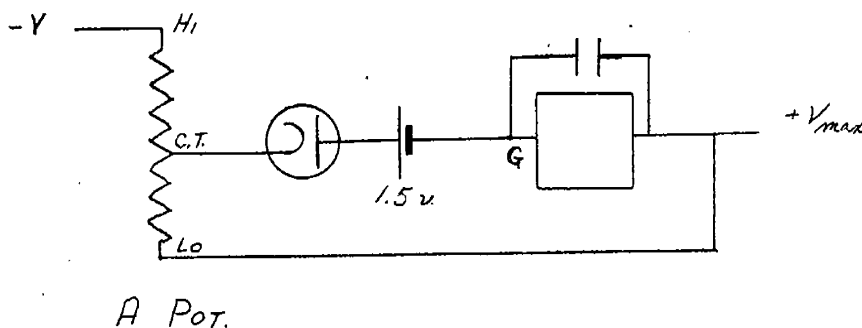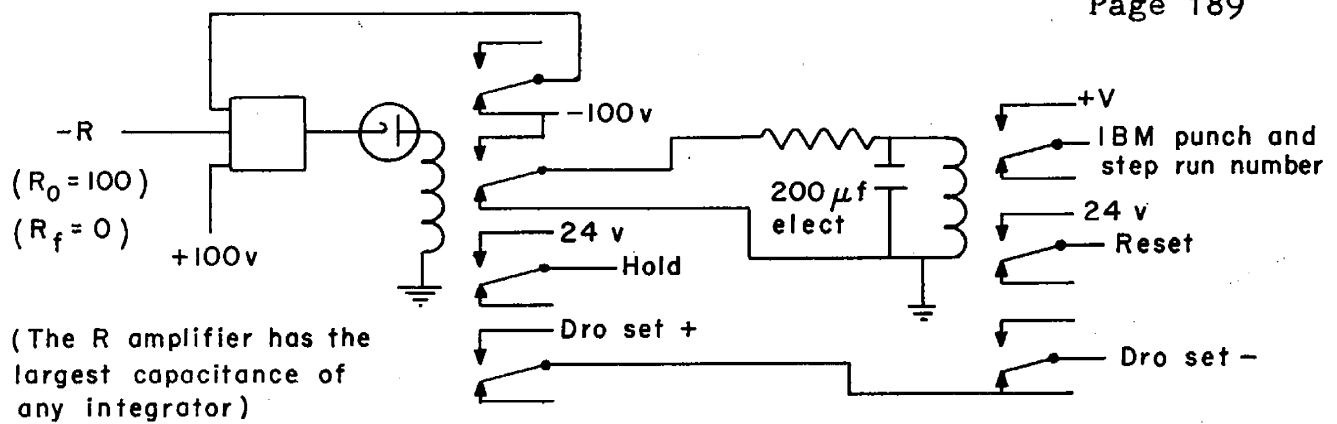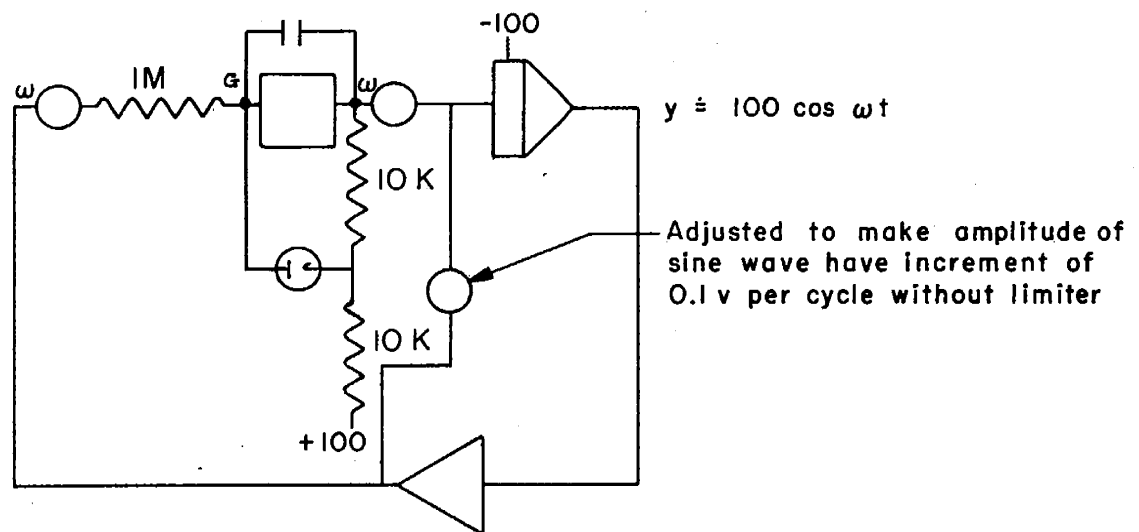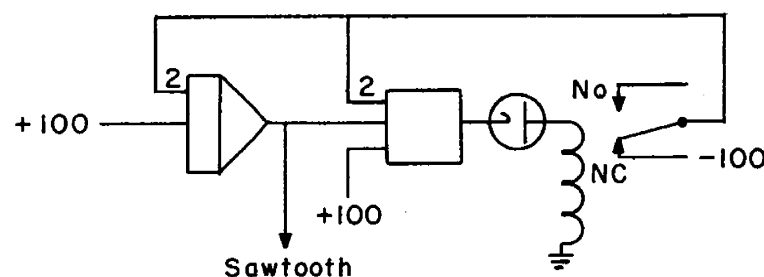

Fig. 91 — Circuit for finding the maximum value of a variable

-R
$(R_0 = 100)$
$(R_f = 0)$
+100v

(The R amplifier has the
largest capacitance of
any integrator)

-100v

200 μf
elect

24 v — Hold

Dro set +

+V — IBM punch and
step run number

24 v — Reset

Dro set −

a   Circuit for self-cycling and punching

ω   IM   G   ω   -100

10 K

10 K

+100

$y \doteq 100 \cos \omega t$

Adjusted to make amplitude of
sine wave have increment of
0.1 v per cycle without limiter

b   Circuit giving constant amplitude sine wave

2   2   No
+100   -100
                    NC
            +100
Sawtooth

c   Sawtooth generator

Fig. 90 ——Circuits used in recent problem